

# Challenges in modelling and analyzing quantitative aspects of a bike-sharing product line

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# Outline

QUANTICOL: A brief introduction

Aim of our research activity

Software Product Line Engineering: Behavioral variability analysis

From single product analysis to product family analysis

Case study: A family of Bike-Sharing Systems (BSS)

BSS: Structural requirements

A tool chain experience

From S.P.L.O.T. to FeatureIDE

From S.P.L.O.T. to ClaferMOO

BSS: Behavioural requirements

Model checking value-passing modal specifications

A value-passing modal process algebra

v-ACTL: A logic to express variability

A tool: Variability Model Checker (VMC)

BSS: Modelling and analysis with VMC

A value-passing modal process algebra

Conclusions and future work

## Aim of our research activity in

- ▶ Develop a framework able to deal with ‘quantitative variability’
- ▶ Provide tools to support this framework with formal verification

### Preliminary results

- ▶ We established a **tool chain** of (academic) tools with different functionalities regarding the analysis of SPL, from feature modelling to product derivation, to the quantitative evaluation of the attributes of products (**structural** variability)
- ▶ We defined a **value-passing** modal process algebra interpreted over Modal Transition Systems (MTS) (**behavioural** variability)
- ▶ We defined a deontic (**variability-aware**) extension of an action-based branching-time modal temporal logic able to handle **data**
- ▶ We extended our Variability Model Checker (VMC) to support this setup

# (Software) Product Line Engineering

*Develop a family of products (product line) using a shared platform or architecture (**commonalities**) and mass customization (**variabilities**)*

Aim: maximize commonalities whilst minimizing cost of variations (i.e., of individual products), thus specifically facilitating (software) reuse in a predictive manner

Variability in terms of **features**:

- ▶ End-user visible pieces of functionality that represent both commonalities (e.g., mandatory, required) and variabilities (e.g., optional, alternative)
- ▶ Only specific combinations of features concern valid products

Complex: *“We always have 126,000,000 different bicycles in store! But only the parts for 1,000. . .”*

# Formal methods in SPLE

Computer-aided analysis of variability models

- ▶ Traditionally: focus on modeling/analyzing structural constraints
- ▶ But: software systems often embedded/distributed/safety-critical
- ▶ Important: model/analyze also behavior (e.g., quality assurance)

Goal: rigorously establish critical requirements of (software) systems ⇒ lift success stories from single product/system engineering to SPLE

Recent approaches to formally model behavioral variability:

- ▶ Variants of UML diagrams (Haugen et al., Jézéquel et al.)
- ▶ Extensions of Petri nets (Clarke et al.)
- ▶ Numerous models with LTS-like semantics: MTS (Fischbein et al., Fantechi et al.), I/O automata (Larsen et al., Lauenroth et al.), CCS/CSP (Gruler et al., Gnesi et al., ter Beek et al., Tribastone), FTS

# Bike-sharing systems (BSS)

⇒ QUANTICOL: A Quantitative Approach to Management and Design of Collective and Adaptive Behaviours

EU FP7-ICT FET-Proactive STREP: 1 April 2013 – 31 March 2017

- ▶ Simple concept: a user arrives at a docking station, pays for a bike, uses it for a while and returns it to a station
- ▶ Multiple benefits: reduction of vehicular traffic (congestion), pollution, energy consumption, etc.
  
- ▶ Docking stations distributed over a city, typically close to other public transportation hubs (e.g. subway and tram stations)
- ▶ (Subscribed) users may rent an available bike and drop it off at any station in the city
- ▶ To improve the efficiency and the user satisfaction of BSS, the load between the different stations may be balanced
  - ▶ incentive schemes (rewards) to change the behaviour of users
  - ▶ efficient (dynamic) **redistribution** of bikes between stations

# Quantitative Business models and Product Line Families

Finding the right BSS for a particular city poses many questions

- ▶ How many and what kind of bikes?
- ▶ How many and what kind of stations and where to place them?
- ▶ Which features (antitheft, maintenance, smart services, etc.)?
- ▶ With or without (dynamic) redistribution?
- ▶ Incentives for users to return bikes to less popular stations?
- ▶ Costs and charging policy (credit card, keycard, etc.)?

How to **evaluate** the various options, **costs/benefits**, improvements and changes in a systematic way?

# A BSS Product Line

In the requirements engineering phase we performed **text mining** on a set of documents describing current BSS to extract a set of **features**: NLP approach based on so-called contrastive analysis to identify commonalities and variabilities from natural language documents

Ferrari, Spagnolo, dell'Orletta © SPLC'13

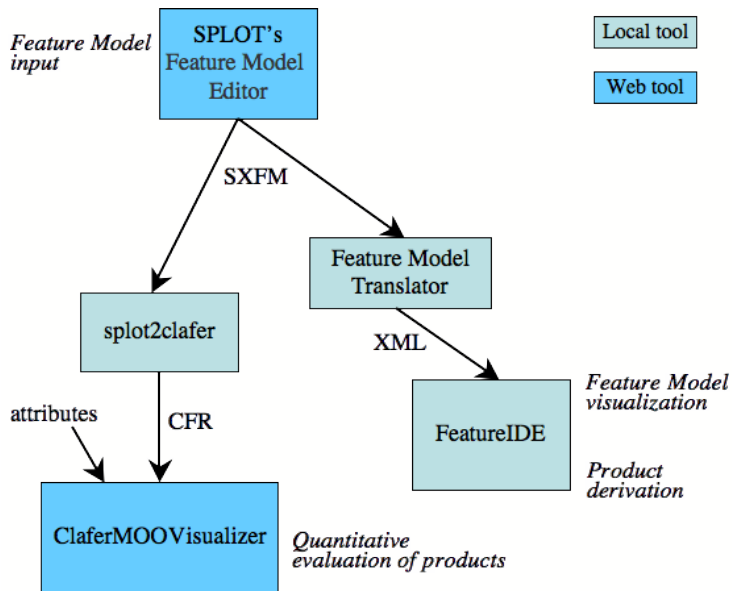
Subsequently we derived an initial feature model from the features that we considered most interesting for developing 'good quality' BSS

We believe they constitute a sufficient starting point for our study

Finally, we sought support in available tools for the possibility of adding **attributes** and **quantitative constraints** to our BSS model



# Tool chain



# Feature model (with S.P.L.O.T.)

Mendonça, Branco, Cowan @ OOPSLA'09

## Feature Diagram

- ▣ Bikesharing
  - ▣ Status
    - ▣ [1..\*]
      - ▣ RTInfoWeb
      - ▣ AllBikesNow
  - ▣ Bike
    - ▣ Localization
      - ▣ [1..\*]
        - ▣ GPS
        - ▣ RFID
      - ▣ Antithieves
    - ▣ DockingStation
      - ▣ [1..1]
        - ▣ Fixed
        - ▣ FixedPortable
        - ▣ Flexible
      - ▣ Maintenance
    - ▣ Redistribution
      - ▣ Reward
    - ▣ Users

## Cross-Tree Constraints

- ▣ (  $\neg$ AllBikesNow  $\vee$  GPS )
- ▣ ( GPS  $\vee$   $\neg$ Antithieves )
- ▣ ( Keycard  $\vee$   $\neg$ KeycardDispenser )
- ▣ (  $\neg$ Keycard  $\vee$  KeycardReader )
- ▣ (  $\neg$ Keycard  $\vee$  KeycardDispenser )

[Click to create a constraint](#)

## Feature Information Table

Id:   
Name:   
Description:   
Type:   
#Children:   
Tree level:

## Feature Model Statistics

#Features	29
#Mandatory	5
#Optional	10
#XOR groups	1
#OR groups	4
#Grouped	13
#Cross-Tree Constraints (CTC)	5
CTCR (%)	0.21
#CTC distinct vars	6
CTC clause density	0.83

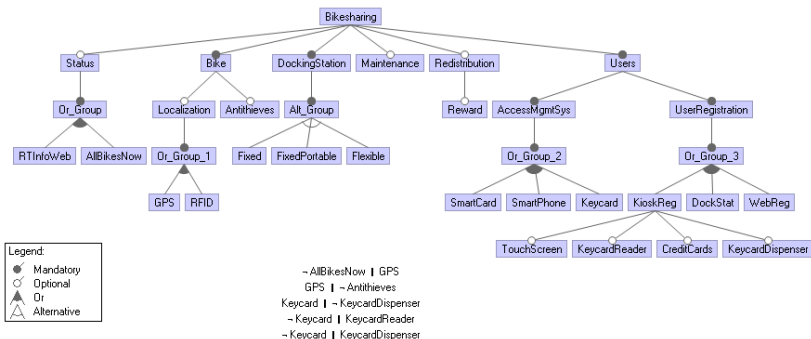
## Feature Model Analysis

✓ Consistency	Consistent
✓ Dead Features	None
✓ Core Features	<u>6 feature(s)</u>
✓ Valid Configurations	60,840

# Feature model (with FeatureIDE)

Thüm, Käßtner, Benduhn, Meinicke, Saake, Leich

© Science of Computer Programming, 2014



We may turn this into an **attributed feature model**, e.g. to measure the "quality" of different products

# Attributed feature model in Clafer

Bąk, Czarnecki, Wąsowski @ SLE'10, Murashkin, Antkiewicz, Rayside, Czarnecki @ SPLC'13

```
abstract Feature
  customersat : integer
  cost : integer
  capacity : integer

abstract SecFeature : Feature
  security : integer

abstract BIKES
  or Status : Feature ?
  [ customersat = 25 ]
  [ cost = 0 ]
  [ capacity = 0 ]
  RTInfoWeb : Feature
  [ customersat = 10 ]
  [ cost = 5 ]
  [ capacity = 0 ]
  AllBikesNow : Feature
  [ customersat = 20 ]
  [ cost = 10 ]
  [ capacity = 0 ]
  Bike : Feature
  [ customersat = 0 ]
  [ cost = 0 ]
  [ capacity = 0 ]

or Localization : Feature ?
  [ customersat = 3 ]
  [ cost = 3 ]
  [ capacity = 0 ]
  RFID : Feature
  [ customersat = 10 ]
  [ cost = 10 ]
  [ capacity = 0 ]
  GPS : Feature
  [ customersat = 15 ]
  [ cost = 15 ]
  [ capacity = 0 ]
  Antithieves : SecFeature ?
  [ customersat = 5 ]
  [ cost = 7 ]
  [ capacity = 0 ]
  [ security = 1 ]
xor DockingStation : SecFeature
  [ customersat = 0 ]
  [ cost = 0 ]
  [ capacity = 0 ]
  [ security = 1 ]
  Fixed : Feature
  [ customersat = 17 ]
  [ cost = 30 ]
  [ capacity = 5 ]

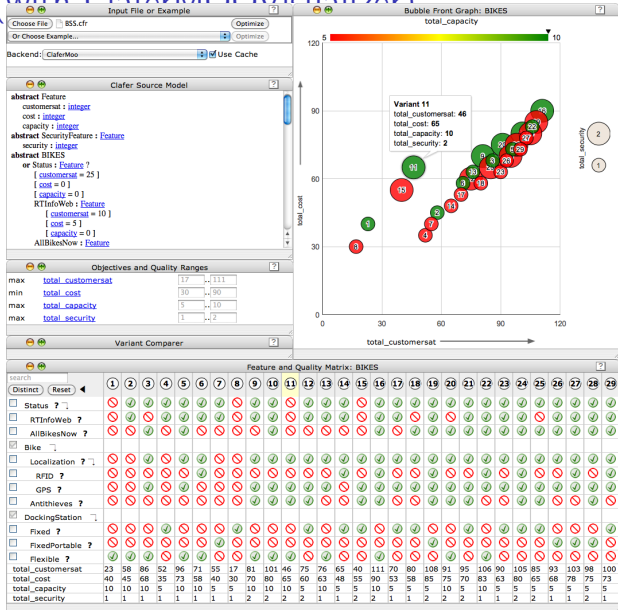
FixedPortable: Feature
  [ customersat = 20 ]
  [ cost = 35 ]
  [ capacity = 5 ]
Flexible: Feature
  [ customersat = 23 ]
  [ cost = 40 ]
  [ capacity = 10 ]
  [ Antithieves => GPS ]
  [ AllBikesNow => GPS ]

total_customersat : integer =
  sum Feature.customersat
total_cost : integer =
  sum Feature.cost
total_capacity : integer =
  sum Feature.capacity
total_security : integer =
  sum SecFeature.security

Mybike : BIKES
<< max Mybike.total_customersat >>
<< min Mybike.total_cost >>
<< max Mybike.total_capacity >>
<< max Mybike.total_security >>
```

# Multi-objective optimization

(with ClaferMOOVisualizer)



# Behavioural requirements of a PL

**Structural requirements** identify the **features** of the different products

**Behavioural requirements** define the **admitted sequences of operations**

We consider a BSS with  $N$  stations and a fleet of  $M$  bikes;  
Each station  $i$  has a capacity  $K_i$ ; Redistribution is **optional**

1. Users arrive at station  $i$
2. If a user arrives at a station with no available bike, (s)he leaves
3. Otherwise, (s)he takes a bike and chooses station  $j$  to return it
4. If there are less than  $K_j$  bikes at station  $j$  when (s)he arrives, (s)he returns the bike and leaves
5. If the station is full she chooses another station  $k$  and goes there
6. A redistribution of bikes **may** be asked for and **may** possibly occur
7. The user rides like this again until (s)he can return the bike

# The framework so far

Aim: develop a framework able to handle behavioural variability and provide tools to support it with formal verification (model checking)

Main ingredient: Modal Transition Systems (MTS)

- ▶ LTS distinguishing admissible **may** and necessary **must** transitions

Larsen, Thomsen @ LICS'88

- ▶ Recognized as a useful model to describe in a compact way the possible **behaviour** of all the products (LTS) of a product family

Fischbein, Uchitel, Braberman @ ROSATEA'06

- ▶ MTS cannot model variability constraints regarding **alternative** features, nor regarding **requires/excludes** inter-feature relations

Asirelli, ter Beek, Fantechi, Gnesi @ iFM'10

- ▶ Our solution: add a set of **variability constraints** to the MTS to be able to decide which derivable products (LTS) are valid ones

Asirelli, ter Beek, Fantechi, Gnesi @ SPLC'11

# Modal Transition Systems (MTS)

Larsen, Thomsen @ LICS'88

A behavioural model, amenable to model checking, able to formalize

1. **shared behaviour**: common among all variants
2. **variation points**: differentiate between variants

A **Labelled Transition System** (LTS) is a quadruple  $(Q, A, \bar{q}, \rightarrow)$  where  $Q$  is a set of states,  $A$  is a set of actions,  $\bar{q} \in Q$  is the initial state and  $\rightarrow \subseteq Q \times A \times Q$  is the transition relation

A **Modal Transition System** (MTS) is a quintuple  $(Q, A, \bar{q}, \rightarrow_{\square}, \rightarrow_{\diamond})$  such that  $(Q, A, \bar{q}, \rightarrow_{\square} \cup \rightarrow_{\diamond})$  is an LTS, called its underlying LTS

An MTS has two distinct transition relations

1. **may** transition relation  $\rightarrow_{\diamond} \subseteq Q \times A \times Q$ : **possible** transitions
2. **must** transition relation  $\rightarrow_{\square} \subseteq Q \times A \times Q$ : **required** transitions

By definition, any required transition is also possible:  $\rightarrow_{\square} \subseteq \rightarrow_{\diamond}$   
 $(\dashrightarrow \equiv \rightarrow_{\diamond} \setminus \rightarrow_{\square})$



## Derive products (implemented in VMC)

A **product LTS** is obtained from a family MTS in the following way

1. include all (reachable) must transitions and
2. include a subset of the (reachable) may transitions

Each selection gives rise to a different variant

Let  $(Q, A, \bar{q}, \delta^\diamond, \delta^\square)$  be an MTS

The set  $\{\mathcal{P}_i = (Q_i, A, \bar{q}, \delta_i) \mid i > 0\}$  of **product LTSs** is obtained by considering each pair of  $Q_i \subseteq Q$  and  $\delta_i \subseteq \delta^\diamond \cup \delta^\square$  to be defined s.t.

1. every  $q \in Q_i$  is reachable in  $\mathcal{P}_i$  from  $\bar{q}$  via transitions from  $\delta_i$  and
2. there exists no  $(q, a, q') \in \delta^\square \setminus \delta_i$  such that  $q \in Q_i$

v-ACTL is used to complement a behavioural description by an MTS by expressing those constraints that MTS cannot model

## v-ACTL: A logic to express variability

v-ACTL defines **action formulas** (boolean compositions of actions, denoted by  $\psi$ ), **state formulas** ( $\phi$ ) and **path formulas** ( $\pi$ )

Let  $a, b \in \mathcal{A}$ . Action formulas are built over a set  $\mathcal{A}$  of actions

$$\psi ::= \text{true} \mid a \mid \neg\psi \mid \psi \wedge \psi$$

( $\text{false} \equiv \neg\text{true}$ ,  $\psi \vee \psi' \equiv \neg(\neg\psi \wedge \neg\psi')$  and  $\psi \implies \psi' \equiv \neg\psi \vee \psi'$ )

The satisfaction relation  $a \models \psi$  of a formula  $\psi$  by  $a$  is

$a \models \text{true}$  always holds

$a \models b$  iff  $a = b$

$a \models \neg\psi$  iff  $a \not\models \psi$

$a \models \psi \wedge \psi'$  iff  $a \models \psi$  and  $a \models \psi'$

# Syntax of v-ACTL

$$\begin{aligned}\phi & ::= \text{true} \mid \neg\phi \mid \phi \wedge \phi \mid [\psi]\phi \mid [\psi]^\square\phi \mid \\ & \quad E\pi \mid A\pi \mid \mu Y.\phi(Y) \mid \nu Y.\phi(Y) \\ \pi & ::= F\phi \mid F^\square\phi \mid F\{\psi\}\phi \mid F^\square\{\psi\}\phi\end{aligned}$$

( $Y$  is a propositional variable,  $\phi(Y)$  is syntactically monotone in  $Y$ )

( $\text{false} \equiv \neg\text{true}$ ,  $\psi \vee \psi' \equiv \neg(\neg\psi \wedge \neg\psi')$  and  $\psi \implies \psi' \equiv \neg\psi \vee \psi'$ )

$\mu$  and  $\nu$ : **recursion** (“finite looping”/“liveness” and “looping”/“safety”)

# Informal semantics of v-ACTL

$[\psi] \phi$  in all next states reachable by a **may** transition executing an action satisfying  $\psi$ ,  $\phi$  holds

$[\psi]^\square \phi$  in all next states reachable by a **must** transition executing an action satisfying  $\psi$ ,  $\phi$  holds

$\langle \psi \rangle \phi \equiv \neg[\psi] \neg\phi$  a next state exists, reachable by a **may** transition executing an action satisfying  $\psi$ , in which  $\phi$  holds

$\langle \psi \rangle^\square \phi \equiv \neg[\psi]^\square \neg\phi$  a next state exists, reachable by a **must** transition executing an action satisfying  $\psi$ , in which  $\phi$  holds

$(\langle \psi \rangle^\square$  and  $[\psi]^\square$  represent the classic **deontic** modalities  $O$  and  $P$ )

# Informal semantics of v-ACTL

A **full path** is a path that cannot be extended further ( $q \cdots$  or  $q \nrightarrow$ )

$E \pi$  there exists a full path on which  $\pi$  holds

$A \pi$  on all possible full paths,  $\pi$  holds

$F \phi$  there exists a future state in which  $\phi$  holds

$F^{\square} \phi$  there exists a future state in which  $\phi$  holds and all transitions until that state are **must** transitions

$F \{ \psi \} \phi$  there exists a future state, reached by an action satisfying  $\psi$ , in which  $\phi$  holds

$F^{\square} \{ \psi \} \phi$  there exists a future state, reached by an action satisfying  $\psi$ , in which  $\phi$  holds and all transitions until that state are **must** transitions

$G \phi \equiv \neg F \neg \phi$  the path is a full path on which  $\phi$  holds in all states

$AG \phi \equiv \neg EF \neg \phi$  in all states on all paths,  $\phi$  holds

## Model checking: $T \models \psi$ ?

Verify a property expressed as a logical formula  $\psi$  over a model  $T$

- ▶ If  $T \not\models \psi$ , then it is usually easy to generate a **counterexample**
- ▶ If  $T$  is finite, model checking thus reduces to a graph search

### On-the-fly model checking

- ▶ Only a fragment of the overall state space might need to be generated and analyzed to be able to produce the correct result
- ▶ Improves performance and allows to handle infinite-state systems

### Bounded model checking:

- ▶ Start evaluation by assuming a certain value as maximum depth
  - ▶ If the evaluation reaches a result within the requested depth, then the result holds for the whole system
  - ▶ Otherwise the maximum depth is increased and the evaluation is retried (preserving all useful partial results)
- ▶ Setting a small initial maximum depth and a small automatic increment of this bound at each (re-)evaluation failure leads to a reasonable (almost minimal) **explanation**

# VMC: Variability Model Checker

ter Beek, Mazzanti, Sulova @ FM'12, ter Beek, Gnesi, Mazzanti @ SPLC'12

VMC builds on optimization of UMC (input: UML state machines)

ter Beek, Fantechi, Gnesi & Mazzanti @ *Science of Computer Programming*, 2011

VMC accepts as input a model specified in the modal process algebra (+ **variability constraints** of form **AL**Ternative, **EX**cludes, **RE**quires?)

- ▶ interactively explore the model (MTS)
- ▶ derive and explore (all) the model's valid variants (LTSSs)
- ▶ visualize the model/variants graphically as MTS/LTSSs
- ▶ verify v-ACTL properties over MTSs/LTSSs
- ▶ interactively explain why a property is (not) satisfied

VMS is freely usable online: <http://fmtlab.isti.cnr.it/vmc/>

Model checking of v-ACTL formulae over MTS can be achieved in a complexity that is **linear** w.r.t. the state space size

## Extending the framework with data

Critical point in the formalization by MTS: lack of a possibility to model an adequate representation of the **data** that may need to be described when considering real systems (even for family of BSS)

Other possible approaches:

- ▶ **Parametric MTS** (Kretinsky, Larsen et al)
- ▶ Parametric modelling with the formal process-algebraic specification language **mCRL2** and its industry-strength toolset (research in progress by ter Beek & de Vink)



# A value-passing modal process algebra

Let  $\mathcal{A}$  be a set of actions, let  $a \in \mathcal{A}$  and let  $L \subseteq \mathcal{A}$

**Processes** are built from terms and actions according to the syntax

$$\begin{aligned} N &::= [P] \\ P &::= K(e) \mid P/L/P \end{aligned}$$

$[P]$  denotes a closed system, i.e. it cannot evolve on input actions  $a(?v)$

$K(e)$  is a process identifier from set of process definitions of form

$$K(v) \stackrel{\text{def}}{=} T$$

$$\begin{aligned} T &::= nil \mid K(e) \mid A.T \mid T + T \mid [e \bowtie e] T \\ A &::= a(e) \mid a(\text{may}, e) \mid a(?v) \mid a(\text{may}, ?v) \\ e &::= v \mid \text{int} \mid e \pm e \end{aligned}$$

$\bowtie \in \{<, \leq, =, \neq, \geq, >\}$ ,  $v$  is a variable, **int** is an integer,

$\pm \in \{+, -, \times, \div\}$

# A value-passing modal process algebra

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$\bowtie \in \{<, \leq, =, \neq, \geq, >\}$ ,  $v$  is a variable,  $\mathbf{int}$  is an integer,

$\pm \in \{+, -, \times, \div\}$

# Processes

*nil* terminated process that has finished execution

*K* process identifier that is used for modelling recursive sequential processes

*A.P* process that can execute action *A* and then behave as *P*

*P + Q* process that can non-deterministically choose to behave as *P* or as *Q*

*P / L / Q* process formed by the parallel composition of *P* and *Q*  
(it can synchronize on actions in *L* and interleave others)

We distinguish **must** actions  $a \in \delta^\square$  and **may but not must** actions  $a(\text{may}) \in \delta^\diamondsetminus \delta^\square$   
(each action type is treated differently in the SOS semantics)

## Value-passing BSS specification

```
Station(I,N,J,M) = request(I).
```

```
  ( [N=0] nobike(I).Station(I,N,J,M) +  
    [N>0] givebike(I).Station(I,N-1,J,M) ) +  
  deliver(I).Station(I,N+1,J,M) +  
  redistribute(may,?FROM,?TO,?K).  
  ( [TO = I] Station(I,N+K,J,M) +  
    [TO /= I] Station(I,N,J,M) ) +  
  [N > M] redistribute(may,I,J,N-M).Station(I,M,J,M)
```

```
net STATIONS = Station(s1,2,s2,2) /redistribute/ Station(s2,2,s1,2)
```

```
Users(I,J) = request(I).
```

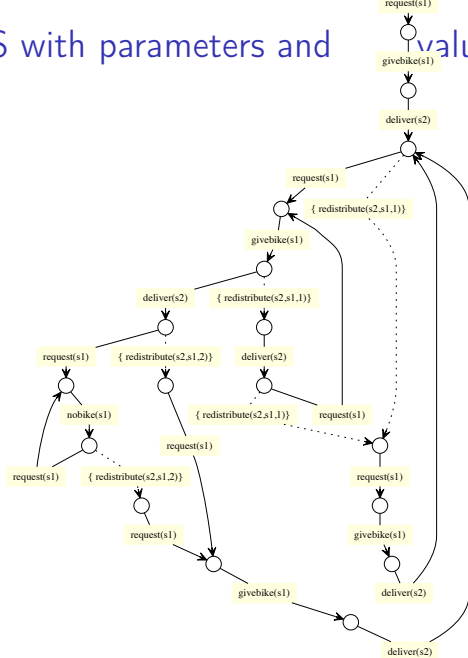
```
  ( givebike(I).deliver(J).Users(I,J) +  
    nobike(I).Users(I,J) )
```

```
net USERS = Users(s1,s2) -- // Users(s2,s1)
```

```
net BSS = STATIONS /request,givebike,nobike,deliver/ USERS
```

# MTS with parameters and

values



## Extending v-ACTL with action values

We only need to extend the definition of action formulas and their satisfaction relation

Let  $a, b \in \mathcal{A}$ . Action formulas are built over a set  $\mathcal{A}$  of *actions*

$$\psi ::= \text{true} \mid a \mid a(e) \mid \neg\psi \mid \psi \wedge \psi$$

Now add to the satisfaction relation  $a \models \psi$  of a formula  $\psi$  by  $a$

$a(e) \models \text{true}$  always holds

$a(e) \models b$  iff  $a = b$

$a(e) \models b(*)$  iff  $a = b$

$a(e) \models b(e')$  iff  $a = b$  and  $e = e'$

$a(e) \models \neg\psi$  iff  $a(e) \not\models \psi$

$a(e) \models \psi \wedge \psi'$  iff  $a(e) \models \psi$  and  $a(e) \models \psi'$

## Extending VMC

The extended modelling and verification environment described so far has been implemented in **VMC v6.0** (November 2013)

`http://fmt.isti.cnr.it/vmc/v6.0`

Accepts models specified in the value-passing modal process algebra

Allows model checking properties expressed in value-passing v-ACTL

# Model checking modal specifications

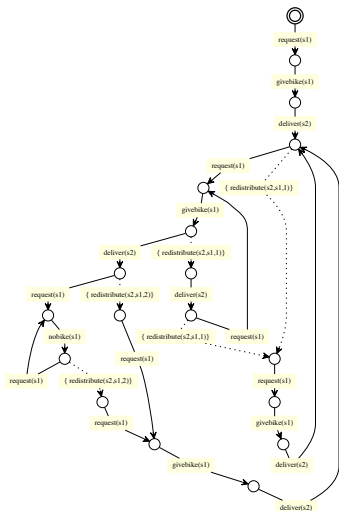
- ▶ **Eventually** it **must** occur that station 1 has no bikes:  $EF^{\square} \{nobike(s1)\}$  true

- ▶ **Eventually** it **may** occur that station 2 has no more bikes:  $EF \{nobike(s2)\}$  true

- ▶ It is **always** the case that **eventually** station 1 **must** give a bike, **possibly after** it has first received bikes **after** redistribution:

$$AG((EF^{\square} \{givebike(s1)\}) \text{ true}) \vee$$

$$(EF^{\square} [redistribute(*,s1,*)] EF^{\square} \{givebike(s1)\} \text{ true})$$



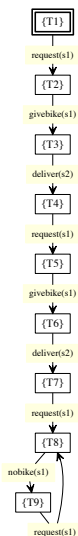


# From modal specifications of families to products

Products may be derived according to an extension of the algorithms already present in VMC, for **example**:

Verification results of v-CTL properties over an MTS are **inherited** by the entire family of derived product according to the following rules:

- ▶ Formulas without negation and only composed from *false*, *true* and the operators  $\wedge$ ,  $\vee$ ,  $\langle \rangle^\square$ ,  $[\ ]$ ,  $\mu$ ,  $\nu$ ,  $EF^\square$ ,  $EF^\square\{\}$ ,  $AF^\square$ ,  $AF^\square\{\}$  and  $AG$  that are valid for a family MTS are also valid for all its product LTSs
- ▶ Formulas without negation and only composed from *false*, *true* and the operators  $\wedge$ ,  $\vee$ ,  $\langle \rangle$ ,  $\mu$ ,  $\nu$ ,  $EF$  and  $EF\{\}$  that are false for a family MTS are false for all its product LTSs



## Future work

Study and implement the **derivation** of products in the presence of both **structural** constraints (ALT, EXC, REQ from feature models) and **quantitative** constraints (attributed feature models)

Study the **inheritance** of the result of verifying a v-ACTL formula over an MTS by its product LTS in the presence of both types of constraints

**Scalability?**