A uniform framework for the operational semantics of quantitative process calculi

Part 1: Introduction to the framework and example(s) of use

R. De Nicola[1], D. Latella[2], M. Loreti[3] M. Massink[2]

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Preamble

- A uniform framework for the operational semantics of quantitative process calculi
  - Part 1:
    Introduction to the framework and example(s) of use.
    R. De Nicola (IMT), D. Latella (CNR-ISTI), M. Loreti (UNIFI) M. Massink (CNR-ISTI)
    [accepted for publication in ACM Computing Surveys]
  - Part 2:
    Bisimulation of State-to-Function Labeled Transition Systems - a coalgebraic view.
    D. Latella (CNR-ISTI), M. Massink (CNR-ISTI), E.-P. de Vink (TU Eindhoven & CWI Amsterdam)
    [ACCAT 2012]

[EU FET Proactive IP ASCENS autonomic service-components ensembles]

Stochastic Process Algebra: *Stochastically Timed* PA

Goal:

Integration of

- *functional* system model descriptions with
- *non-functional*, e.g. performance/dependability, ones

in a single mathematical framework

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  - action *durations*, (or *delays* before instantaneous actions)
- Combining
  - Labeled Transition Systems (LTS) with
  - Continuous Time Markov Chains (CTMC)

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- the definition of the rate associated to *synchronisations*
- the implementation of the *race condition* principle of CTMC
  - multi-relations *[e.g. PEPA, IML]*
  - proved transition systems *[e.g. TIPP, SπC]*
  - LTS with numbered transitions *[e.g. LCTMC]*
  - unique rate names *[e.g. StoKLAIM]*
  - rated transition systems / rate transition systems *[e.g. MarCaSPiS]*

Stochastic Process Algebra: Troubles

- Transition multiplicity (race condition)
  - The technicalities set up for dealing with transition multiplicity often blur the conceptual understanding of the calculus.
  - The transition multi-relation defined as the *least multi-relation* induced by a set of SOS rules (unintentionally!) boils down to a *relation*.
- Interaction paradigm and synchronisation rate
  - Use of classical SOS for CCS-like interaction in combination with the *minimal apparent rate* principle may lead to *loss of associativity* for the parallel composition operator.

Labelled State to Function Transition Systems for the definition of Stochastic Process Languages

Labelled State to Function Transition Systems for the definition of Quantitative Process Languages

Timeline

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- *Total, Deterministic, Finite Support FuTSs;*
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- *Total, Deterministic, Finite Support FuTSs;*
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- FuTSs Coalgebraically; Functors $\mathcal{FS}(\cdot, \mathbb{C})$ and $\mathcal{FS}(\cdot, \mathbb{C})^{\mathcal{L}}$;
- Functors induced by FuTSs;
- Bisimilarity results;
- Application to SPCs;
- Related/Future Work.

Simple total deterministic FuTSs

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- countable set of states S
- total deterministic transition relation $\rightarrowtail \subseteq S \times \mathcal{L} \times \mathcal{FS}(\, S, \mathbb{C} \,)$
    - for all $s \in S, \ell \in \mathcal{L} : s \overset{\ell}{\rightarrowtail} u$ for some u
    - if $s \overset{\ell}{\rightarrowtail} v$ and $s \overset{\ell}{\rightarrowtail} u$ then $u = v$ ($\rightarrowtail$ is a function)

with $\mathcal{FS}(\, S, \mathbb{C} \,)$: all total functions $f : S \to \mathbb{C}$ of finite support
(Finite Support FuTSs)

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Operators on $\mathcal{FS}(S, \mathbb{C})$

For all $s, s_1, \ldots, s_m \in S$, $S' \subseteq S$, $\gamma_1, \ldots, \gamma_m \in \mathbb{C}$, v, v_1, v_2 in $\mathcal{FS}(S, \mathbb{C})$:

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For injective binary operator $\bullet : S \times S \to S$ let us define binary operator $\bullet : \mathcal{FS}(\,S,\mathbb{C}\,) \times \mathcal{FS}(\,S,\mathbb{C}\,) \to \mathcal{FS}(\,S,\mathbb{C}\,)$ with:

$$(v_1 \bullet v_2)(s) = \begin{cases} v_1(s_1) \cdot_{\mathbb{C}} v_2(s_2), & \text{if } s = s_1 \bullet s_2, \text{for some } s_1, s_2 \\ 0_{\mathbb{C}} & \text{otherwise} \end{cases}$$

General total deterministic FuTSs

sets of labels $\mathcal{L}_i$, semirings $\mathbb{C}_i$, $(i = 1..n)$

general total deterministic *FuTS* $\mathcal{R} = (\, S, \langle \rightarrowtail_i \rangle_{i=1}^{n} \,)$

- countable set of states S
- total deterministic transition relations $\rightarrowtail_i \subseteq S \times \mathcal{L}_i \times \mathcal{FS}(\, S, \mathbb{C}_i \,)$

PEPA fragment

Given action set $\mathcal{A}$

Hillston (1996)

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Grammar G_{PEPA} is

$P \ ::= \mathbf{nil} \mid (a, \lambda).P \mid P + P \mid P \underset{A}{\bowtie} P \mid X$ of PEPA processes we consider, where $a \in \mathcal{A}$ and $\lambda \in \mathbb{R}_{>0}$.

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Label set $\Delta^{e}_{\mathcal{A}} = \{ \delta^{e}_{a} \mid a \in \mathcal{A} \}$ for semantics

Hillston (1996)

Standard vs. FuTS-semantics

standard semantics $\rightarrow \subseteq PEPA \times (\mathcal{A} \times \mathbb{R}_{\geqslant 0}) \times PEPA$

FuTS-semantics $\rightarrowtail \subseteq PEPA \times \Delta^{e}_{\mathcal{A}} \times \mathcal{FS}(\, PEPA, \mathbb{R}_{\geqslant 0}\,)$

Standard vs. FuTS-semantics

standard semantics $\rightarrow \subseteq PEPA \times (\mathcal{A} \times \mathbb{R}_{\geqslant 0}) \times PEPA$

no rule for **nil** $\qquad$ (RAPF) $\dfrac{}{(a,\lambda).P \xrightarrow{a,\lambda} P}$

FuTS-semantics $\rightarrowtail \subseteq PEPA \times \Delta^{e}_{\mathcal{A}} \times \mathcal{FS}(\, PEPA, \mathbb{R}_{\geqslant 0}\,)$

(NIL) $\dfrac{}{\mathbf{nil} \overset{\delta^e_a}{\rightarrowtail} [\,]_{\mathbb{R}_{\geqslant 0}}}$ $\qquad$ (RAPF1) $\dfrac{}{(a,\lambda).P \overset{\delta^e_a}{\rightarrowtail} [\,P \mapsto \lambda\,]}$ $\qquad$ (RAPF2) $\dfrac{b \neq a}{(a,\lambda).P \overset{\delta^e_b}{\rightarrowtail} [\,]_{\mathbb{R}_{\geqslant 0}}}$

$[\,]_{\mathbb{R}_{\geqslant 0}}(\,Q\,) = 0$ all Q and $[\,R \mapsto \lambda\,](\,Q\,) = \lambda$ for $Q = R$ and 0 otherwise

Standard vs. FuTS-semantics

standard semantics

$$\text{(CNS)}\ \frac{P \xrightarrow{a,\lambda} P' \quad X := P}{X \xrightarrow{a,\lambda} P'} \quad \text{(CHO1)}\ \frac{P \xrightarrow{a,\lambda} P'}{P + Q \xrightarrow{a,\lambda} P'} \quad \text{(CHO2)}\ \frac{Q \xrightarrow{a,\lambda} Q'}{P + Q \xrightarrow{a,\lambda} P'}$$

FuTS-semantics

$$\text{(CNS)}\ \frac{P \overset{\delta_a^e}{\rightarrowtail}_p \mathcal{P} \quad X := P}{X \overset{\delta_a^e}{\rightarrowtail}_p \mathcal{P}} \quad \text{(CHO)}\ \frac{P \overset{\delta_a^e}{\rightarrowtail} \mathcal{P} \quad Q \overset{\delta_a^e}{\rightarrowtail} \mathcal{Q}}{P + Q \overset{\delta_a^e}{\rightarrowtail} \mathcal{P} + \mathcal{Q}}$$

$$(v_1 + v_2)(R) = v_1(R) + v_2(R)$$

Choice and Transition Multiplicity

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- $R_1 = R_2 = R$ and $\lambda_1 = \lambda_2 = \lambda$:
  $(a, \lambda).R + (a, \lambda).R \xrightarrow{\delta_a^e} [R \mapsto 2 \cdot \lambda]$

Standard vs. FuTS-semantics

standard semantics

$$(\text{PAR2}) \; \frac{P \xrightarrow{a,\lambda_1} P' \quad Q \xrightarrow{a,\lambda_2} Q' \quad a \in A}{P \underset{A}{\bowtie} Q \xrightarrow{a,\lambda} P' \underset{A}{\bowtie} Q'} \qquad \lambda = \lambda_1 \cdot \lambda_2 \cdot \frac{\min\{r_a(P), r_a(Q)\}}{r_a(P) \cdot r_a(Q)}$$

FuTS-semantics

$$(\text{PAR2}) \; \frac{P \overset{\delta_a^e}{\rightarrowtail} \mathcal{P} \quad Q \overset{\delta_a^e}{\rightarrowtail} \mathcal{Q} \quad a \in A}{P \underset{A}{\bowtie} Q \overset{\delta_a^e}{\rightarrowtail} (\mathcal{P} \underset{A}{\bowtie} \mathcal{Q}) \cdot \frac{\min\{\oplus\mathcal{P}, \oplus\mathcal{Q}\}}{\oplus\mathcal{P} \cdot \oplus\mathcal{Q}}}$$

$$(v_1 \underset{A}{\bowtie} v_2)(R) = \begin{cases} v_1(R_1) \cdot v_2(R_2), & \text{if } R = R_1 \underset{A}{\bowtie} R_2 \\ 0, & \text{otherwise} \end{cases} \qquad \oplus v = \sum_{R \in PEPA} v(R)$$

Standard vs. FuTS-semantics

standard semantics

$$(\text{PAR1a})\ \frac{P \xrightarrow{a,\lambda} P' \quad a \notin A}{P \underset{A}{\bowtie} Q \xrightarrow{a,\lambda} P' \underset{A}{\bowtie} Q} \qquad (\text{PAR1b})\ \frac{Q \xrightarrow{a,\lambda} Q' \quad a \notin A}{P \underset{A}{\bowtie} Q \xrightarrow{a,\lambda} P \underset{A}{\bowtie} Q'}$$

FuTS-semantics

$$(\text{PAR1})\ \frac{P \overset{\delta_a^e}{\rightarrowtail} \mathcal{P} \quad Q \overset{\delta_a^e}{\rightarrowtail} \mathcal{Q} \quad a \notin A}{P \underset{A}{\bowtie} Q \overset{\delta_a^e}{\rightarrowtail} (\,\mathcal{P} \underset{A}{\bowtie} \mathcal{X}_Q\,) + (\,\mathcal{X}_P \underset{A}{\bowtie} \mathcal{Q}\,)}$$

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Application

The approach has been applied for the (re-)definition of the semantics of significant fragments of

- TIPP [Hermanns, H., Herzog, U., and Mertsiotakis, V. 1998]
- PEPA [Hillston 1996]
- EMPA [Aldini A., Bernardo M., Corradini F. 2010]
- Stochastic CCS [Klin & Sassone 2008], [De Nicola et a. 2009]
- Stochastic π-calculus [Priami, 1995]
- StoKLAIM [De Nicola et al. 2005,9]
- MarCaSPiS [De Nicola et al. 2008]
- IML [Hermanns 2002]
- Language for Markov Automata (Non-deterministic FuTSs). [Eisentraut et al. 2010]

The relationship with the original definitions (correctness) has been proven.

(Some) Related Work

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  Based on *Rated TS* a subclass of FuTSs. Approach more 'syntax oriented': Rate computation "distributed" among rules, whereas functions in the FuTS approach are computed in a more direct way, *within* each rule. S-/W-GSOS guarantee stochastic bisimilarity is a congruence;

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  General framework. Axiomatic. Based on μCRL;
- [Bernardo et al. 2010]
  ULTRAS: similar to FuTSs; function co-domains: not required to be semi-rings, but ordered sets.
- . . . more in the paper!

Conclusions & Future Work

FuTSs approach

- Effective, compositional, exploitation of operators on continuations $\mathcal{P}$
- Simple and elegant solution to the
  - transition *multiplicity*, and
  - (CCS-)parallel composition associativity

  issues.
- Dealing with *weights* (e.g. PEPA, EMPA *passive actions*)
- Dealing with quantitative, non-deterministic, probabilistic PCs (e.g. IML, MAL)

Currently and in the Future:

- Completing the coalgebraic view;
- Applying the framework to CINA languages;
- Investigating equivalences other than bisimilarity;
- Further study w.r.t. the use of *deterministic* FuTSs for MAL.

Thank You!