Analysis of Trust and Reputation Systems

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Context and Motivation

Trust and Reputation Systems: decision support tools used to drive parties' interactions on the basis of parties' reputation.

Examples: eBay, TripAdvisor, Amazon, iTunes Store, Android Store, ...

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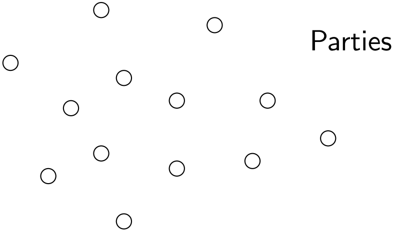
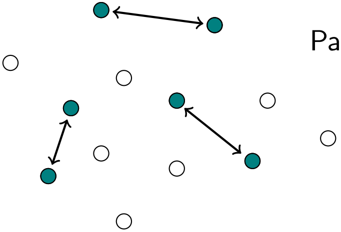
Goal: to assess confidence in the decisions calculated by trust and reputation systems

Example: A Generic Trust and Reputation System



Example: A Generic Trust and Reputation System

Interactions: parties in a trust and reputation system are free to interact.



Parties

Example: A Generic Trust and Reputation System

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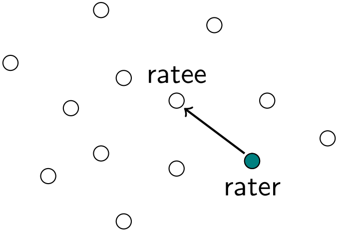
Rating Values: after each interaction parties rate each other.



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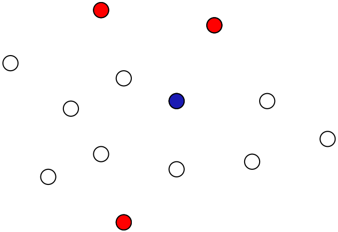

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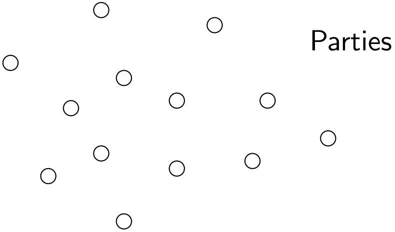

Reputation: aggregate ratings are used to compute reputation scores for a given party.

Computational Trust: parties' trustworthiness is evaluated on the basis of parties' past behaviours.

Probabilistic Trust

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- Set of $n + 1$ values denoting service's quality, $\mathscr{O} = \{0, 1, ..., n\}$

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Goal: the task of computing reputation scores boils down to inferring the *true* distribution's parameters for a given party.

Context and Motivation

Principal questions

- How do we quantify the *confidence* in the decisions calculated by the system?
- How is this confidence related to such parameters as *decision strategy* and *number of available ratings*?
- Is there an optimal strategy that maximizes confidence as more and more information becomes available?

In order to answer these questions, we are interested in:

- a general framework to analyse probabilistic trust systems based on **bayesian decision theory**
- **loss functions** for evaluating decisions' consequences, $\mathrm{L}(\cdot, \cdot)$
- **expected** and **worst-case loss**, respectively $r^n(\cdot, \cdot)$ and $w^n(\cdot)$, for quantifying confidence in the systems
- expressions for the **limit value** as $n \to \infty$ and the **rate** of convergence

Outline

1. Formal Set Up
   - Evaluation of Decision Functions
2. Results
3. Examples
4. Conclusions

Formal Set Up

Observation framework: describes how observations are probabilistically generated.

Observation framework: $\mathscr{S} = (\mathscr{O}, \Theta, \mathscr{F}, \pi(\cdot))$

- $\mathscr{O}$ is a finite non-empty set of *observations*
- Θ is a set of *world states*, or *parameters*
- $\mathscr{F} = \{p(\cdot|\theta)\}_{\theta \in \Theta}$ is a set of probability distributions on $\mathscr{O}$ indexed by Θ
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Assumption: the sequence $o^n = (o_1, ..., o_n)$ is a realization of a random vector $O^n = (O_1, ...O_n)$, where the r.v. O_i's are i.i.d. given $\theta \in \Theta$

Formal Set Up

Example: $\mathscr{S} = (\mathscr{O}, \Theta, \mathscr{F}, \pi(\cdot))$

- Set of binary outcomes, representing *success* and *failure*, $\mathscr{O} = \{o, \bar{o}\}$
- Bernoulli distribution, $\mathscr{B}(\theta)$

$$p(o|\theta) = \theta \quad \text{and} \quad p(\bar{o}|\theta) = 1 - \theta$$

- Set of parameters $\Theta \subseteq (0, 1)$

Formal Set Up

Example: $\mathscr{S} = (\mathscr{O}, \Theta, \mathscr{F}, \pi(\cdot))$

- Set of $n+1$ values representing service's quality, $\mathscr{O} = \{0, 1, ..., n\}$
- Binomial distribution, $\mathscr{B}in(n, \theta)$

$$p(o|\theta) = \binom{n}{o} \theta^o (1-\theta)^{n-o}$$

- Set of parameters $\Theta \subseteq (0, 1)$

Formal Set Up

Decision framework: $\mathscr{DF} = (\mathscr{S}, \mathscr{D}, \mathrm{L}(\cdot,\cdot), \{g^{(n)}\}_{n\geq 1})$

- $\mathscr{S} = (\mathscr{O}, \Theta, \mathscr{F}, \pi(\cdot))$ is an observation framework
- $\mathscr{D}$ is a decision set
- $\mathrm{L}(\cdot,\cdot) = \Theta \times \mathscr{D} \to \mathbb{R}^+$ is a loss function
- $\{g^{(n)}\}_{n\geq 1}$ is a family of decision functions, one for each $n \geq 1$, $g^{(n)} : \mathscr{O}^n \to \mathscr{D}$

Decision framework: describes how decisions are taken.

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Loss functions: evaluate the consequences of possible decisions associating a loss to each decision.

$\mathrm{L}(\theta, d)$ quantifies the **loss incurred** when making a decision $d \in \mathscr{D}$, given that the real behaviour of the party is $\theta \in \Theta$.

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Decision functions: formalise the decision-making process.

Two main types of decisions

- Evaluate party's behaviour (reputation).
- Predict the outcome of the next interaction.

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Reputation framework $\rightarrow \mathscr{D} = \Theta$

Prediction framework $\rightarrow \mathscr{D} = \mathscr{O}$

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Examples

ML, $g^{(\mathtt{ML})}(o^n) = \arg\min_\theta D(t_{o^n} || p(\cdot|\theta))$

MAP, $g^{(\mathtt{MAP})}(o^n) = \theta$ implies $p(\theta|o^n) \geq p(\theta'|o^n)$ for each $\theta' \in \Theta$

Evaluation of Decision Functions

Frequentist risk: for a parameter $\theta \in \Theta$, the frequentist risk associated to a decision function g after n observation is just the expected loss computed over $\mathscr{O}^n$,

$$R^n(\theta, g) = \sum_{o^n \in \mathscr{O}^n} p(o^n|\theta)\mathrm{L}(\theta, g(o^n)).$$

Expected loss for a fixed behaviour θ after n observations

Evaluation of Decision Functions

Bayes risk: is the expected value of the risk $R^n(\theta, g)$, computed with respect to the a priori distribution $\pi(\cdot)$,

$$r^n(\pi, g) = \mathbb{E}_\pi[R^n(\theta, g)] = \sum_\theta \pi(\theta) R^n(\theta, g).$$

The *minimum bayes risk* is defined as $r^* = \sum_\Theta \pi(\theta) \mathrm{L}(\theta, d_\theta)$.

Expected loss considering user's belief over possible behaviours

Evaluation of Decision Functions

Worst risk: is the maximum risk $R^n(\theta, g)$ over possible parameters $\theta \in \Theta$,

$$w^n(g) = \max_{\theta \in \Theta} R^n(\theta, g).$$

The *minimum worst risk* is defined as $w^* = \max_{\theta \in \Theta} \mathrm{L}(\theta, d_\theta)$

Maximum expected loss over all possible behaviours

Evaluation of Decision Functions

Limit values: we study the behaviour of bayes and worst risk when an increasing number of ratings is available ($n \to \infty$).

Exponential convergence: limit values for both risks are achievable exponentially fast ($2^{-n\rho}$).

Rate: the exponent ρ determine how fast the limit is approached.

Some results

Best achievable rate: (for any decision function) the upper bound is the least Chernoff Information

Theorem

if $\lim r^n(\pi, g) = r^*$ then

$$\mathrm{rate}(r^n(\pi, g)) \leq \underbrace{\min_{\theta \neq \theta'} C(p_\theta, p_{\theta'})}_{\text{least Chernoff Information}}$$

Similarly for the worst risks w^n and w^*.

Some results

Asymptotically optimal: both MAP and ML are asymptotically optimal decision functions

Theorem: g either MAP or ML

$$\lim_n r^n = r^* \qquad \text{and} \qquad \mathrm{rate}(r^n) = \min_{\theta \neq \theta'} C(p_\theta, p_{\theta'})$$

Similarly for w^n.

Example 1: System assessment

Peers' behaviour: Bernoulli distribution $\mathscr{B}(\theta)$ over the set $\mathscr{O} = \{0, 1\}$.

Parameters set: Θ is a discrete set of N points $0 < \gamma, 2\gamma, ..., N\gamma < 1$, for a positive parameter γ.

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Parameters set: Θ is a discrete set of N points $0 < \gamma, 2\gamma, ..., N\gamma < 1$, for a positive parameter γ.

Loss function: $\mathrm{L}(\theta, \theta') = ||p(\cdot|\theta) - p(\cdot|\theta')||_1$.

Decision function: g is a ML reputation function.

Priori distribution: uniform distribution $\pi(\cdot)$ over Θ.

Example 1: System assessment

Goals:

- Study the rate of convergence of the risk functions depending on γ.
- Compare the analytical approximations of the risk functions with the empirical values.

$$r^n \approx r^* + 2^{-nR} \text{ and } w^n \approx w^* + 2^{-nR}$$

where $R = \min_{\theta \neq \theta'} C(p_\theta, p_{\theta'})$

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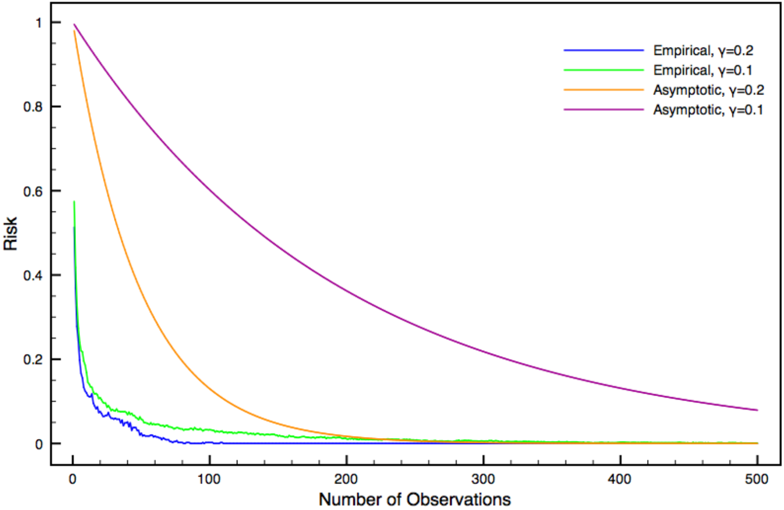


Intuition: for large values of γ, the incurred loss will be exactly zero. For small values of γ, the incurred loss will be small but nonzero in most cases.

Conclusions

- We proposed a framework based on bayesian decision theory to analyse trust and reputation systems
- We examinated the behaviour of two risk quantities: bayes and worst risks to quantify confidency in system's decisions.

Our results allow to characterize the asymptotic behaviour of probabilistic trust systems :

- showing how to determine limits value of both bayes and worst risks, and their exact exponential rates of convergence
- showing that ML and MAP decision functions are asymptotically optimal

More on the analysis of trust and reputation systems

Joint work with Rocco De Nicola and Francesco Tiezzi

Stochastic analysis

StoKlaim: stochastic extension of Klaim

$\rightarrow$ specification of trust and reputation systems

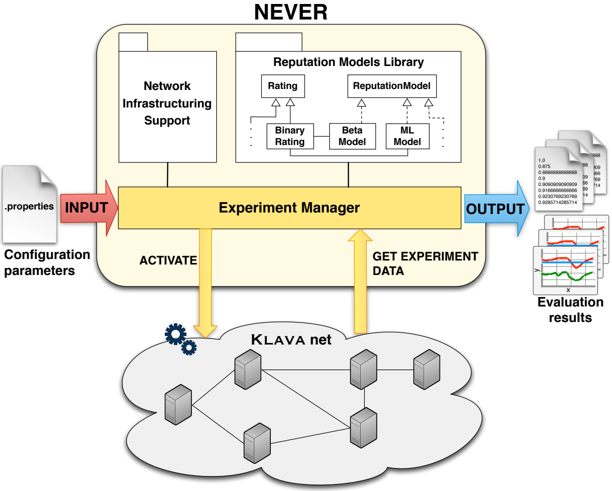
MoSL: stochastic logic

$\rightarrow$ formally express the desired properties

SAM: tool that supports the analysis of StoKlaim specifications

$\rightarrow$ check the properties of interest

More on the analysis of trust and reputation systems



NEVER : Network-aware Evaluation Environment for Reputation Systems

Thank you for your attention