The ULTraS Framework: New Behavioral Equivalences for Nondeterministic and Probabilistic Processes and Their Spectrum

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Uniform Behavioral Models and Equivalences

- Behavioral models of complex interacting systems are mostly based on labeled transition systems [Keller, 1976].
- Behavioral equivalences studied in the 1980's to establish a connection between different LTS models that exhibit the same behavior.
- Several generalizations to deal with probabilistic and/or timed systems since the late 1980's, yielding different models and equivalences.
- Can we provide a uniform definition of the various models/equivalences?
- Do new models/equivalences with interesting properties emerge?
- Taking inspiration from two extensions of the LTS model:
  - *Simple probabilistic automata* [Segala, 1995].
  - *Rate transition systems* [De Nicola-Latella-Loreti-Massink, 2009].

The ULTRAS Model

- $(D, \sqsubseteq_D, \bot_D)$: preordered set equipped with a minimum denoted by $\bot_D$, representing one-step reachability.
- A uniform labeled transition system on $(D, \sqsubseteq_D, \bot_D)$, or D-ULTRAS, is a triple $\mathcal{U} = (S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times [S \to D]$ is a transition relation.
- $\mathcal{U}$ is *functional* iff $\longrightarrow$ is a function from $S \times A$ to $[S \to D]$.
- Given a transition $s \xrightarrow{a} \mathcal{D}$, function $\mathcal{D}$ represents the distribution of reachability of all possible states from s via that transition.
- If $\mathcal{D}(s') = \bot_D$, then s' is not reachable from s via that transition.

Encoding Specific Models

- An LTS can be encoded as a functional $\mathbb{B}$-ULTRAS, where $\mathbb{B} = \{\bot, \top\}$ is the support set of the Boolean algebra with $\bot \sqsubseteq_{\mathbb{B}} \top$.
- An LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times S$ is a transition relation.
- *External and internal nondeterminism.*
- Corresponding functional $\mathbb{B}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \mathcal{D}_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\mathcal{D}_{s,a}(s') = \begin{cases} \top & \text{if } s \xrightarrow{a} s' \\ \bot & \text{if } (s, a, s') \notin \longrightarrow \end{cases}$ for all $s' \in S$.

- A GPLTS (or action-labeled discrete-time Markov chain – ADTMC) can be encoded as a functional $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.

- A generative probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{(0,1]} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,p_1} s'$ and $s \xrightarrow{a,p_2} s'$, then $p_1 = p_2$.
  - $\sum\{\!| p \in \mathbb{R}_{(0,1]} \mid \exists a \in A. \exists s' \in S. s \xrightarrow{a,p} s' |\!\} \in \{0, 1\}$ for all $s \in S$.

- *External and internal probabilistic choices.*

- Corresponding functional $\mathbb{R}_{[0,1]}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \mathcal{D}_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\mathcal{D}_{s,a}(s') = \begin{cases} p & \text{if } s \xrightarrow{a,p} s' \\ 0 & \text{if } \nexists p \in \mathbb{R}_{(0,1]}. s \xrightarrow{a,p} s' \end{cases}$ for all $s' \in S$.

- An RPLTS (or discrete-time Markov decision process – MDP) can be encoded as a functional $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.
- A reactive probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{(0,1]} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,p_1} s'$ and $s \xrightarrow{a,p_2} s'$, then $p_1 = p_2$.
  - $\sum\{\!| p \in \mathbb{R}_{(0,1]} \mid \exists s' \in S.\, s \xrightarrow{a,p} s' |\!\} \in \{0, 1\}$ for all $s \in S$ and $a \in A$.
- *External nondeterminism & internal probabilistic choices.*
- Corresponding functional $\mathbb{R}_{[0,1]}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \mathcal{D}_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\mathcal{D}_{s,a}(s') = \begin{cases} p & \text{if } s \xrightarrow{a,p} s' \\ 0 & \text{if } \nexists p \in \mathbb{R}_{(0,1]}.\, s \xrightarrow{a,p} s' \end{cases}$ for all $s' \in S$.

- An NPLTS (which is an MDP with internal nondeterminism) can be encoded as an $\mathbb{R}_{[0,1]}$-ULTRAS with the usual $\leq$.
- A nondeterministic and probabilistic LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{[0,1]}]$ is a transition relation.
  - $\sum_{s' \in S} \mathcal{D}(s') = 1$ for all $s \xrightarrow{a} \mathcal{D}$.
- *External/internal nondeterminism & internal probabilistic choices.*
- The corresponding $\mathbb{R}_{[0,1]}$-ULTRAS is $(S, A, \longrightarrow)$ itself.
- Not functional due to the coexistence of internal nondeterminism and probabilistic choices.

- A GMLTS (or action-labeled continuous-time Markov chain – ACTMC) can be encoded as a functional $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.
- A generative Markovian LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{>0} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,\lambda_1} s'$ and $s \xrightarrow{a,\lambda_2} s'$, then $\lambda_1 = \lambda_2$.
- *External and internal rate-based probabilistic choices.*
- Corresponding functional $\mathbb{R}_{\geq 0}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \mathcal{D}_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\mathcal{D}_{s,a}(s') = \begin{cases} \lambda & \text{if } s \xrightarrow{a,\lambda} s' \\ 0 & \text{if } \nexists \lambda \in \mathbb{R}_{>0}.\, s \xrightarrow{a,\lambda} s' \end{cases}$ for all $s' \in S$.

- An RMLTS (or continuous-time Markov decision process – CTMDP) can be encoded as a functional $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.
- A reactive Markovian LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times \mathbb{R}_{>0} \times S$ is a transition relation.
  - Whenever $s \xrightarrow{a,\lambda_1} s'$ and $s \xrightarrow{a,\lambda_2} s'$, then $\lambda_1 = \lambda_2$.
- *External nondeterminism & internal rate-based probabilistic choices.*
- Corresponding functional $\mathbb{R}_{\geq 0}$-ULTRAS $\mathcal{U} = (S, A, \longrightarrow_{\mathcal{U}})$:
  - $s \xrightarrow{a}_{\mathcal{U}} \mathcal{D}_{s,a}$ for all $s \in S$ and $a \in A$.
  - $\mathcal{D}_{s,a}(s') = \begin{cases} \lambda & \text{if } s \xrightarrow{a,\lambda} s' \\ 0 & \text{if } \nexists \lambda \in \mathbb{R}_{>0}.\, s \xrightarrow{a,\lambda} s' \end{cases}$ for all $s' \in S$.

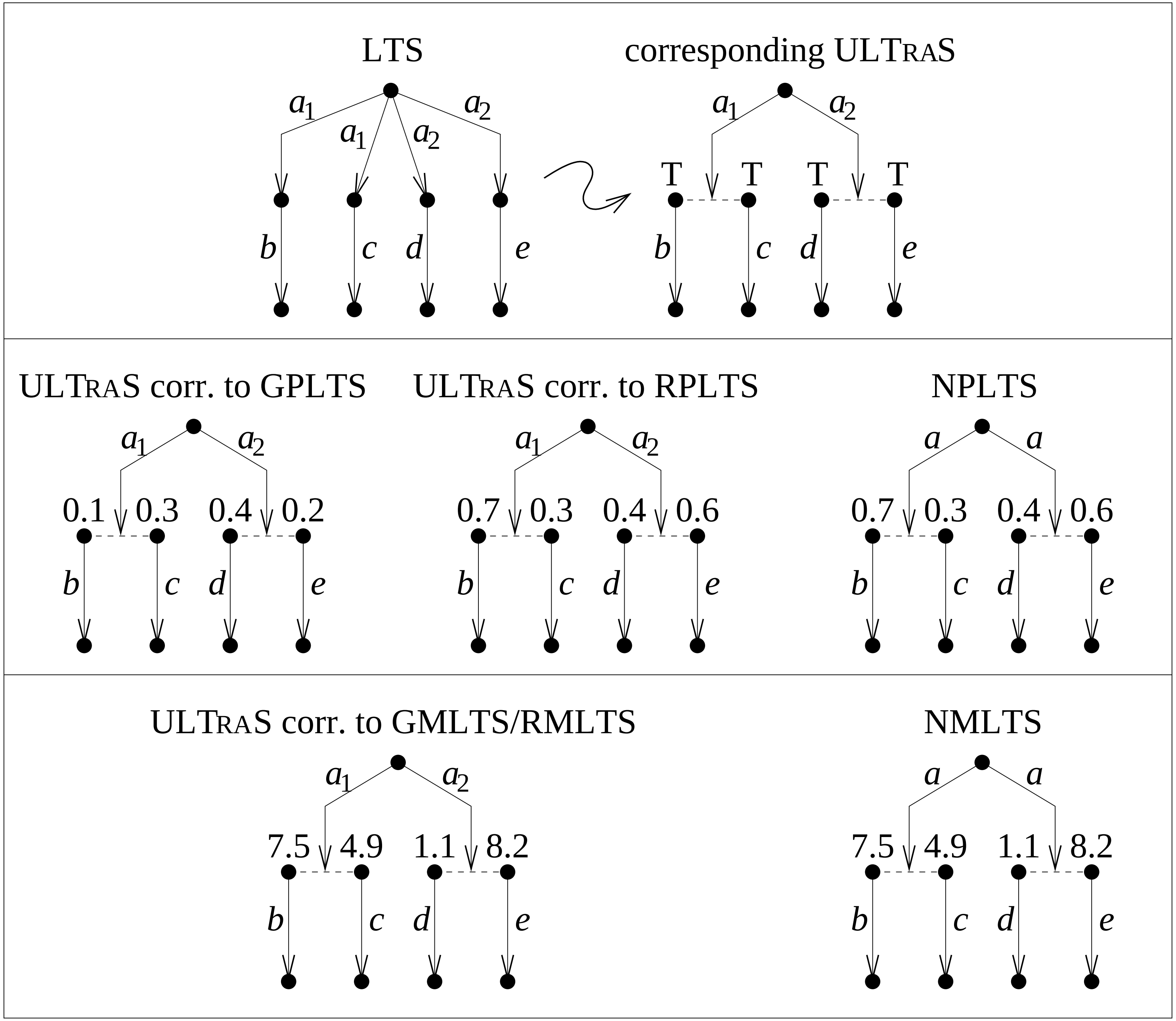
- An NMLTS (which is a CTMDP with internal nondeterminism) can be encoded as an $\mathbb{R}_{\geq 0}$-ULTRAS with the usual $\leq$.

- A nondeterministic and Markovian LTS is a triple $(S, A, \longrightarrow)$ where:
  - S is an at most countable set of states.
  - A is a countable set of transition-labeling actions.
  - $\longrightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{\geq 0}]$ is a transition relation.
  - $\sum_{s' \in S} \mathcal{D}(s') > 0$ for all $s \xrightarrow{a} \mathcal{D}$.

- *Ext./int. nondeterminism & internal rate-based probabilistic choices.*

- The corresponding $\mathbb{R}_{\geq 0}$-ULTRAS is $(S, A, \longrightarrow)$ itself.

- Not functional due to the coexistence of internal nondeterminism and rate-based probabilistic choices.


LTS
corresponding ULTRAS
a_1 a_1 a_2 a_2
T T T T
b c d e
ULTRAS corr. to GPLTS
0.1 0.3 0.4 0.2
ULTRAS corr. to RPLTS
0.7 0.3 0.4 0.6
NPLTS
a a
0.7 0.3 0.4 0.6
ULTRAS corr. to GMLTS/RMLTS
7.5 4.9 1.1 8.2
NMLTS
a a
7.5 4.9 1.1 8.2

Behavioral Equivalences for the ULTRAS Model

- $(M, \sqsubseteq_M, \bot_M)$: preordered set equipped with a minimum denoted by $\bot_M$, representing multi-step reachability.

- A measure function on $(M, \sqsubseteq_M, \bot_M)$ for $\mathcal{U} = (S, A, \longrightarrow)$, or M-measure function for $\mathcal{U}$, is a function $\mathcal{M}_M : S \times A^* \times 2^S \to M$ such that the value of $\mathcal{M}_M(s, \alpha, S')$ is defined by induction on $|\alpha|$ and depends only on the reachability of a state in S' from state s through computations labeled with trace α.

- A measure function somehow subsumes the existence of two operators:
  - A *multiplicative operator* $\otimes$ that combines into an M-value the D-values corresponding to each individual step along a single computation labeled with trace α that goes from s to S'.
  - An *additive operator* $\oplus$ that combines the M-values computed for each considered computation with the previous operator.

- D and M are not necessarily the same set.
- A D-value $\mathcal{D}(s')$ is related to one-step reachability.
- An M-value $\mathcal{M}_M(s, \alpha, S')$ is related to multi-step reachability.
- Testing equivalence for LTS models: the M-value will be a pair of $\mathbb{B}$-values – *rather than a single* $\mathbb{B}$*-value* – to take into account the possibility and the necessity of reaching S' from s after α.
- Equivalences for NPLTS models: the M-value will be a nonemtpy set of $\mathbb{R}_{[0,1]}$-values – *rather than a single* $\mathbb{R}_{[0,1]}$*-value* – to take into account all possible ways of resolving internal nondeterminism.
- Equivalences for Markovian models: the M-value will be an $\mathbb{R}_{[0,1]}$-valued function – *rather than a single* $\mathbb{R}_{\geq 0}$*-value* – representing for each possible end-to-end/step-by-step deadline the probability (or set of probabilities) of reaching S' from s via α within the considered deadline.

- Let $\mathcal{U} = (S, A, \longrightarrow)$ be a D-ULTRAS.
- Let $\mathcal{M}_M$ be an M-measure function for $\mathcal{U}$.
- Strong equivalences: no abstraction from invisible actions.
- An equivalence relation $\mathcal{B}$ over S is an $\mathcal{M}_M$-bisimulation iff, whenever $(s_1, s_2) \in \mathcal{B}$, then for all actions $a \in A$ and groups of equivalence classes $\mathcal{G} \in 2^{S/\mathcal{B}}$:
$$\mathcal{M}_M(s_1, a, \bigcup \mathcal{G}) = \mathcal{M}_M(s_2, a, \bigcup \mathcal{G})$$
We say that $s_1, s_2 \in S$ are $\mathcal{M}_M$-bisimilar, written $s_1 \sim_{\mathrm{B},\mathcal{M}_M} s_2$, iff there exists an $\mathcal{M}_M$-bisimulation $\mathcal{B}$ over S such that $(s_1, s_2) \in \mathcal{B}$.
- We say that $s_1, s_2 \in S$ are $\mathcal{M}_M$-trace equivalent, written $s_1 \sim_{\mathrm{Tr},\mathcal{M}_M} s_2$, iff for all traces $\alpha \in A^*$:
$$\mathcal{M}_M(s_1, \alpha, S) = \mathcal{M}_M(s_2, \alpha, S)$$

- A D-observation system is a D-ULTRAS $\mathcal{O} = (O, A, \longrightarrow_{\mathcal{O}})$ where O contains a distinguished success state denoted by ω such that, whenever $\omega \xrightarrow{a} \mathcal{D}$, then $\mathcal{D}(o) = \bot_D$ for all $o \in O$.
- D-valued function δ for the interaction system $\mathcal{I}(\mathcal{U}, \mathcal{O})$ to combine the target distributions of the synchronizing transitions of $\mathcal{U}$ and $\mathcal{O}$, which preserves $\bot_D$ and is injective.
- States are configurations (s, o) that are successful when $o = \omega$: $\mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O})$.
- We say that $s_1, s_2 \in S$ are $\mathcal{M}_M^{\delta}$-testing equivalent, written $s_1 \sim_{\mathrm{Te}, \mathcal{M}_M^{\delta}} s_2$, iff for every D-observation system $\mathcal{O} = (O, A, \longrightarrow_{\mathcal{O}})$ with initial state $o \in O$ and for all traces $\alpha \in A^*$:
$$\mathcal{M}_M^{\delta,\mathcal{O}}((s_1, o), \alpha, \mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O})) = \mathcal{M}_M^{\delta,\mathcal{O}}((s_2, o), \alpha, \mathcal{S}^{\delta}(\mathcal{U}, \mathcal{O}))$$

Retrieving Specific Behavioral Equivalences

- Most of the bisimulation, trace, and testing equivalences appeared in the literature since the 1980's are captured by our general framework ...
- ... except when internal nondeterminism and probability/stochasticity coexist in the considered model.
- For NPLTS models, we have obtained equivalences different from those appeared in the literature, which possess interesting properties.
- For NMLTS models, there are no equivalences defined in the literature, hence we have provided them for the first time.

- Nondeterministic behavioral equivalences:

LTS	$\sim_{\mathrm{B}}$ [Park, 1981][Milner, 1984] $\sim_{\mathrm{Tr}}$ [Brookes-Hoare-Roscoe, 1984] $\sim_{\mathrm{Te}}$ [De Nicola-Hennessy, 1984]	$\sim_{\mathrm{B},\mathcal{M}_{\mathbb{B},\vee}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{B},\vee}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{LC}}_{\mathbb{B}\times\mathbb{B}}}$	functional $\mathbb{B}$-ULTRAS

- Nondeterministic measure functions:

$$\mathcal{M}_{\mathbb{B},\vee}(s,\alpha,S') = \begin{cases} \bigvee\limits_{s'\in S \text{ s.t. } \mathcal{D}_{s,a}(s')\neq\bot} \mathcal{M}_{\mathbb{B},\vee}(s',\alpha',S') & \alpha = a\circ\alpha' \\ \top;\bot & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathbb{B},\wedge}(s,\alpha,S') = \begin{cases} \bigwedge\limits_{s'\in S \text{ s.t. } \mathcal{D}_{s,a}(s')\neq\bot} \mathcal{M}_{\mathbb{B},\wedge}(s',\alpha',S') & \alpha = a\circ\alpha' \\ \top;\bot & \alpha=\varepsilon,\ s\in S'? \end{cases}$$

$$\mathcal{M}_{\mathbb{B}\times\mathbb{B}}(s,\alpha,S') = (\mathcal{M}_{\mathbb{B},\vee}(s,\alpha,S'),\mathcal{M}_{\mathbb{B},\wedge}(s,\alpha,S'))$$

- Probabilistic behavioral equivalences:

GPLTS	$\sim_{\mathrm{PB}}$ $\sim_{\mathrm{PTr}}$ $\sim_{\mathrm{PTe}}$	$\sim_{\mathrm{B},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{NPM}}_{\mathbb{R}_{[0,1]}}}$	functional $\mathbb{R}_{[0,1]}$-ULTraS such that for all $s \in S$ $\sum_{a \in A} \sum_{s' \in S} \mathcal{D}_{s,a}(s') \in \{0, 1\}$
RPLTS	$\sim_{\mathrm{PB}}$ [Larsen-Skou, 1991] $\sim_{\mathrm{PTr}}$ $\sim_{\mathrm{PTe}}$	$\sim_{\mathrm{B},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{\mathbb{R}_{[0,1]}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{PM}}_{\mathbb{R}_{[0,1]}}}$	functional $\mathbb{R}_{[0,1]}$-ULTraS such that for all $s \in S$ and $a \in A$ $\sum_{s' \in S} \mathcal{D}_{s,a}(s') \in \{0, 1\}$
NPLTS	$\sim_{\mathrm{PB,N}}$ $\sim_{\mathrm{PTr,N}}$ $\sim_{\mathrm{PTe,N}}$	$\sim_{\mathrm{B},\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}}$ $\sim_{\mathrm{Tr},\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}}}$ $\sim_{\mathrm{Te},\mathcal{M}^{\mathrm{PM}}_{2^{\mathbb{R}_{[0,1]}}}}$	$\mathbb{R}_{[0,1]}$-ULTraS such that for all $s \xrightarrow{a} \mathcal{D}$ $\sum_{s' \in S} \mathcal{D}(s') = 1$

- Probabilistic measure functions:

$\mathcal{M}_{\mathbb{R}_{[0,1]}}(s, \alpha, S')$	$=$	$\sum_{s' \in S} \mathcal{D}_{s,a}(s') \cdot \mathcal{M}_{\mathbb{R}_{[0,1]}}(s', \alpha', S')$	$\alpha = a \circ \alpha'$
		$1; 0$	$\alpha = \varepsilon,\ s \in S'?$
$\mathcal{M}_{2^{\mathbb{R}_{[0,1]}}_{\cdot}}(s, \alpha, S')$	$=$	$\bigcup_{s \xrightarrow{a} \mathcal{D}} \{ \sum_{s' \in S} \mathcal{D}(s') \cdot p_{s'} \mid p_{s'} \in \mathcal{M}_{2^{\mathbb{R}_{[0,1]}}_{\cdot}}(s', \alpha', S') \}$	$\alpha = a \circ \alpha'$
		$\{1\}; \{0\}$	$\alpha = \varepsilon,\ s \in S'?$

- Markovian behavioral equivalences:

GMLTS	$\sim_{\rm MB}$ [Hillston, 1996] $\sim_{\rm MTr,ete}$ $\sim_{\rm MTr,sbs}$ $\sim_{\rm MTe,ete}$ $\sim_{\rm MTe,sbs}$	$\sim_{{\rm B},\mathcal{M}_{\rm ete}}$ $\sim_{{\rm B},\mathcal{M}_{\rm sbs}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm ete}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm sbs}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm ete}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm sbs}}$	functional $\mathbb{R}_{\geq 0}$-ULTraS
RMLTS	$\sim_{\rm MB}$ $\sim_{\rm MTr,ete,R}$ $\sim_{\rm MTr,sbs,R}$ $\sim_{\rm MTe,ete,R}$ $\sim_{\rm MTe,sbs,R}$	$\sim_{{\rm B},\mathcal{M}_{\rm ete,R}}$ $\sim_{{\rm B},\mathcal{M}_{\rm sbs,R}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm ete,R}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm sbs,R}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm ete,R}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm sbs,R}}$	functional $\mathbb{R}_{\geq 0}$-ULTraS
NMLTS	$\sim_{\rm MB,N}$ $\sim_{\rm MTr,ete,N}$ $\sim_{\rm MTr,sbs,N}$ $\sim_{\rm MTe,ete,N}$ $\sim_{\rm MTe,sbs,N}$	$\sim_{{\rm B},\mathcal{M}_{\rm ete,N}}$ $\sim_{{\rm B},\mathcal{M}_{\rm sbs,N}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm ete,N}}$ $\sim_{{\rm Tr},\mathcal{M}_{\rm sbs,N}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm ete,N}}$ $\sim_{{\rm Te},\mathcal{M}^{\rm RM}_{\rm sbs,N}}$	$\mathbb{R}_{\geq 0}$-ULTraS such that for all $s \xrightarrow{a} \mathcal{D}$ $\sum_{s' \in S} \mathcal{D}(s') > 0$

- Markovian measure functions:

$\mathcal{M}_{\mathrm{ete},\mathrm{R}}(s,\alpha,S')(t)$	$= \begin{cases} \int_0^t \mathrm{E}_a(s) \cdot \mathrm{e}^{-\mathrm{E}_a(s)\cdot x} \cdot \sum_{s' \in S} \frac{\mathcal{D}_{s,a}(s')}{\mathrm{E}_a(s)} \cdot \mathcal{M}_{\mathrm{ete},\mathrm{R}}(s',\alpha',S')(t-x)\,\mathrm{d}x & \alpha = a \circ \alpha',\ \mathrm{E}_a(s) > 0 \\ 1; 0 & \alpha = \varepsilon,\ s \in S'? \end{cases}$
$\mathcal{M}_{\mathrm{sbs},\mathrm{R}}(s,\alpha,S')(\theta)$	$= \begin{cases} (1 - \mathrm{e}^{-\mathrm{E}_a(s)\cdot t}) \cdot \sum_{s' \in S} \frac{\mathcal{D}_{s,a}(s')}{\mathrm{E}_a(s)} \cdot \mathcal{M}_{\mathrm{sbs},\mathrm{R}}(s',\alpha',S')(\theta') & \alpha = a \circ \alpha',\ \theta = t \circ \theta',\ \mathrm{E}_a(s) > 0 \\ 1; 0 & \alpha = \varepsilon,\ s \in S'? \end{cases}$
$\mathcal{M}_{\mathrm{ete},\mathrm{N}}(s,\alpha,S')(t)$	$= \begin{cases} \bigcup_{s \xrightarrow{a} \mathcal{D}} \{\int_0^t \mathcal{D}(S) \cdot \mathrm{e}^{-\mathcal{D}(S)\cdot x} \cdot \sum_{s' \in S} \frac{\mathcal{D}(s')}{\mathcal{D}(S)} \cdot p_{s'}\,\mathrm{d}x \mid p_{s'} \in \mathcal{M}_{\mathrm{ete},\mathrm{N}}(s',\alpha',S')(t-x)\} & \alpha = a \circ \alpha' \\ \{1\}; \{0\} & \alpha = \varepsilon,\ s \in S'? \end{cases}$
$\mathcal{M}_{\mathrm{sbs},\mathrm{N}}(s,\alpha,S')(\theta)$	$= \begin{cases} \bigcup_{s \xrightarrow{a} \mathcal{D}} \{(1 - \mathrm{e}^{-\mathcal{D}(S)\cdot t}) \cdot \sum_{s' \in S} \frac{\mathcal{D}(s')}{\mathcal{D}(S)} \cdot p_{s'} \mid p_{s'} \in \mathcal{M}_{\mathrm{sbs},\mathrm{N}}(s',\alpha',S')(\theta')\} & \alpha = a \circ \alpha',\ \theta = t \circ \theta' \\ \{1\}; \{0\} & \alpha = \varepsilon,\ s \in S'? \end{cases}$

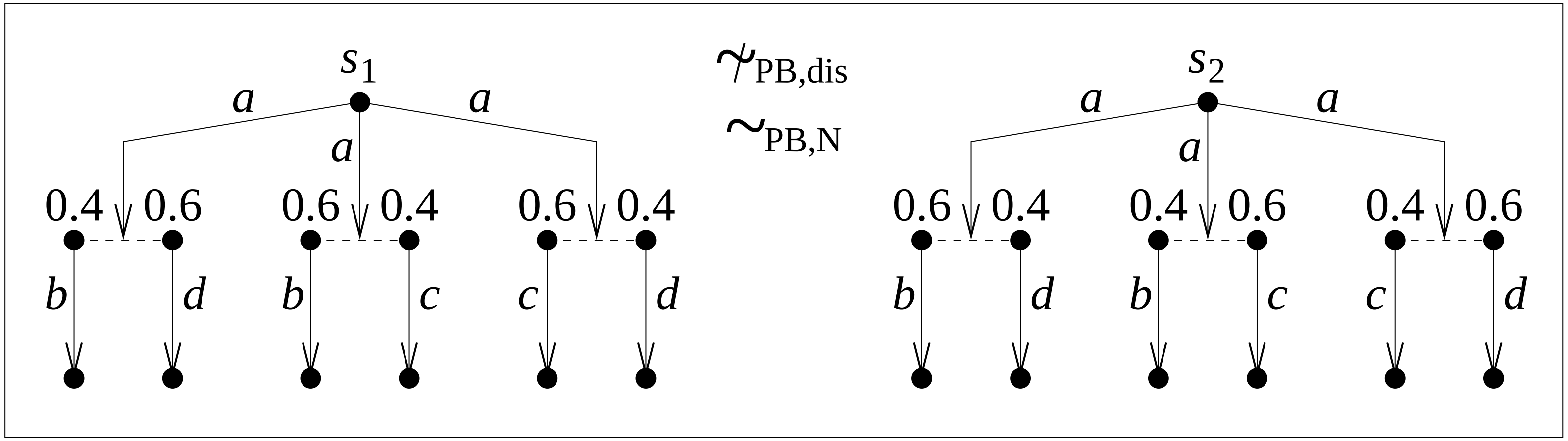
New Behavioral Equivalences for NPLTS Models

- Bisimilarity $\sim_{\mathrm{PB,dis}}$ introduced in [Segala-Lynch, 1994].

- An equivalence relation $\mathcal{B}$ over S is a probabilistic group-distribution bisimulation iff, whenever $(s_1, s_2) \in \mathcal{B}$, then for each $s_1 \xrightarrow{a} \mathcal{D}_1$ there exists $s_2 \xrightarrow{a} \mathcal{D}_2$ such that for all $\mathcal{G} \in 2^{S/\mathcal{B}}$ it holds that $\mathcal{D}_1(\bigcup \mathcal{G}) = \mathcal{D}_2(\bigcup \mathcal{G})$.

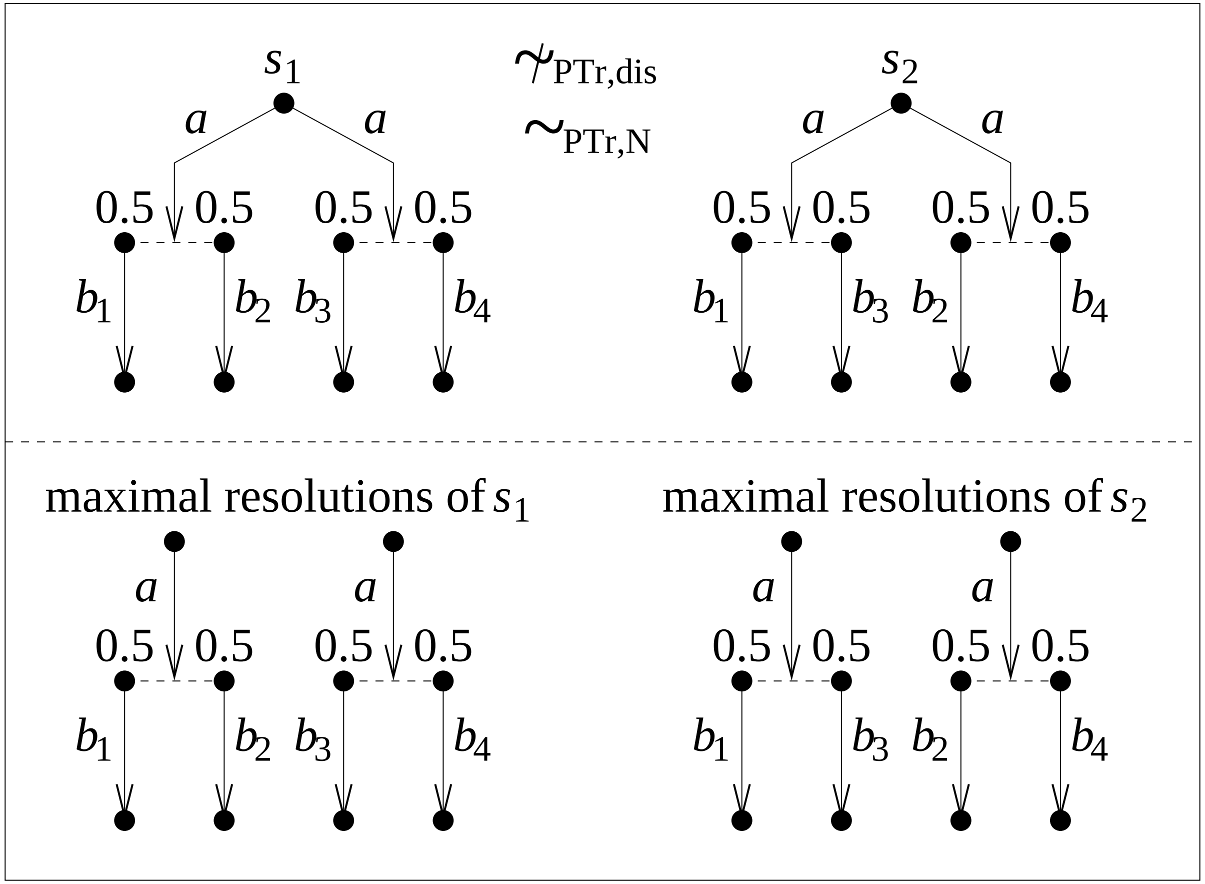
- Our bisimilarity $\sim_{\mathrm{PB,N}}$ is characterized by PML!

- An equivalence relation $\mathcal{B}$ over S is a probabilistic bisimulation iff, whenever $(s_1, s_2) \in \mathcal{B}$, then for all $\mathcal{G} \in 2^{S/\mathcal{B}}$ it holds that for each $s_1 \xrightarrow{a} \mathcal{D}_1$ there exists $s_2 \xrightarrow{a} \mathcal{D}_2$ such that $\mathcal{D}_1(\bigcup \mathcal{G}) = \mathcal{D}_2(\bigcup \mathcal{G})$.



s_1
$\not\sim_{PB,dis}$
$\sim_{PB,N}$
s_2
a
a
a
0.4
0.6
0.6
0.4
0.6
0.4
b
d
b
c
c
d
0.6
0.4
0.4
0.6
0.4
0.6

- Trace equivalence $\sim_{\mathrm{PTr,dis}}$ introduced in [Segala, 1995].
- $s_1 \sim_{\mathrm{PTr,dis}} s_2$ iff for each $\mathcal{Z}_1 \in \mathit{Res}(s_1)$ there exists $\mathcal{Z}_2 \in \mathit{Res}(s_2)$ such that for all $\alpha \in A^*$:
$$\mathit{prob}(\mathcal{CC}(z_{s_1}, \alpha)) = \mathit{prob}(\mathcal{CC}(z_{s_2}, \alpha))$$
and symmetrically for each $\mathcal{Z}_2 \in \mathit{Res}(s_2)$.
- Our trace equivalence $\sim_{\mathrm{PTr,N}}$ is compositional!
- $s_1 \sim_{\mathrm{PTr,N}} s_2$ iff for all $\alpha \in A^*$ it holds that for each $\mathcal{Z}_1 \in \mathit{Res}(s_1)$ there exists $\mathcal{Z}_2 \in \mathit{Res}(s_2)$ such that:
$$\mathit{prob}(\mathcal{CC}(z_{s_1}, \alpha)) = \mathit{prob}(\mathcal{CC}(z_{s_2}, \alpha))$$
and symmetrically for each $\mathcal{Z}_2 \in \mathit{Res}(s_2)$.



s_1
$\not\sim_{\mathrm{PTr,dis}}$
$\sim_{\mathrm{PTr,N}}$
s_2
a
0.5
b_1 b_2 b_3 b_4
maximal resolutions of s_1
maximal resolutions of s_2

- Testing equivalence $\sim_{\mathrm{PTe},\sqcup\sqcap}$ introduced in [Yi-Larsen, 1992] and then studied in [Jonsson-Yi, 1995], [Segala, 1996], and [Deng-Van Glabbeek-Hennessy-Morgan, 2008].

- $s_1 \sim_{\mathrm{PTe},\sqcup\sqcap} s_2$ iff for every NPT $\mathcal{T} = (O, A, \longrightarrow_{\mathcal{T}})$ with initial state $o \in O$:

$$\bigsqcup_{\mathcal{Z}_1 \in Res_{\max}(s_1,o)} prob(\mathcal{SC}(z_{s_1,o})) \;=\; \bigsqcup_{\mathcal{Z}_2 \in Res_{\max}(s_2,o)} prob(\mathcal{SC}(z_{s_2,o}))$$

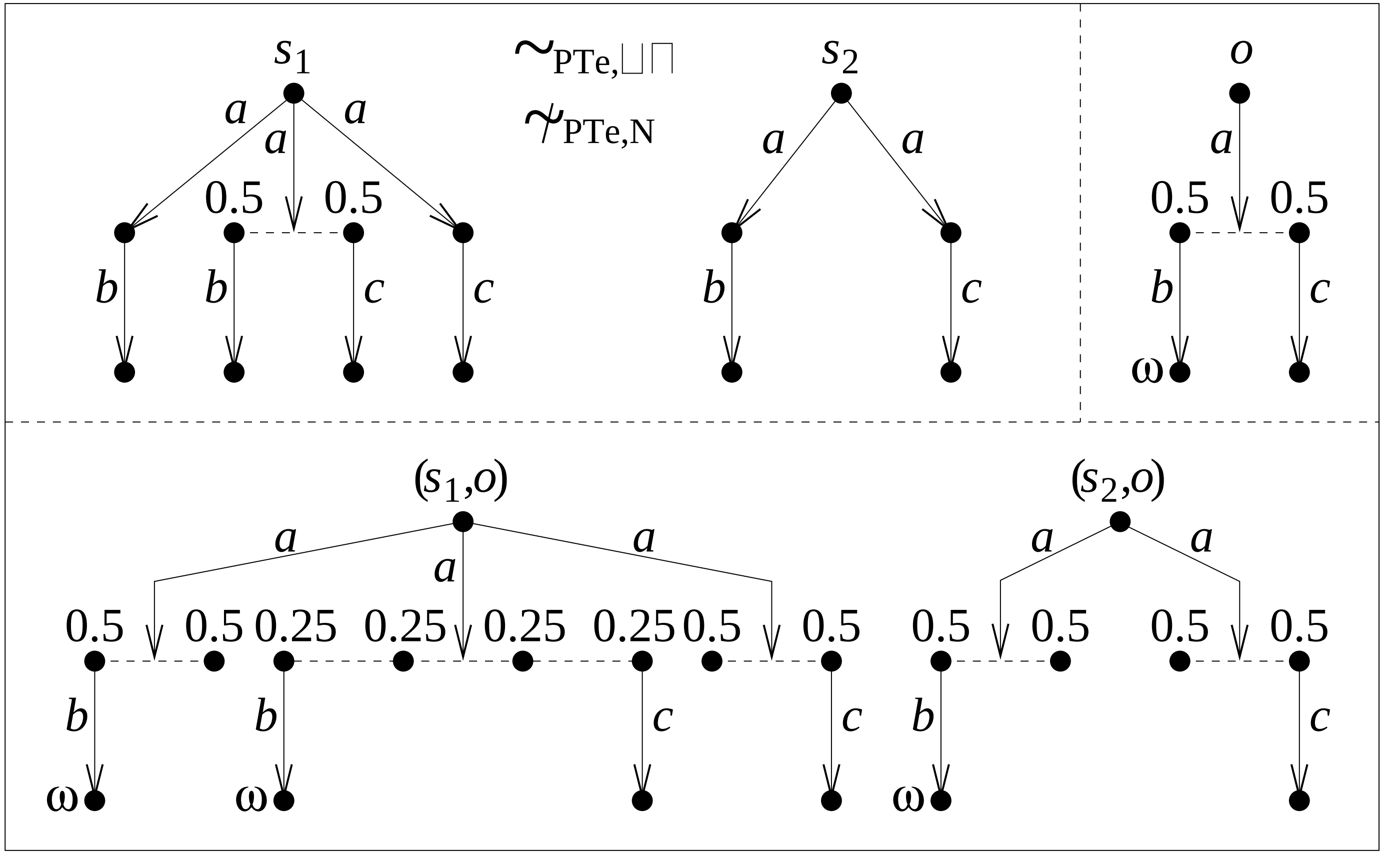
$$\bigsqcap_{\mathcal{Z}_1 \in Res_{\max}(s_1,o)} prob(\mathcal{SC}(z_{s_1,o})) \;=\; \bigsqcap_{\mathcal{Z}_2 \in Res_{\max}(s_2,o)} prob(\mathcal{SC}(z_{s_2,o}))$$

- Our testing equivalence $\sim_{\mathrm{PTe,N}}$ is compatible with the classical one!

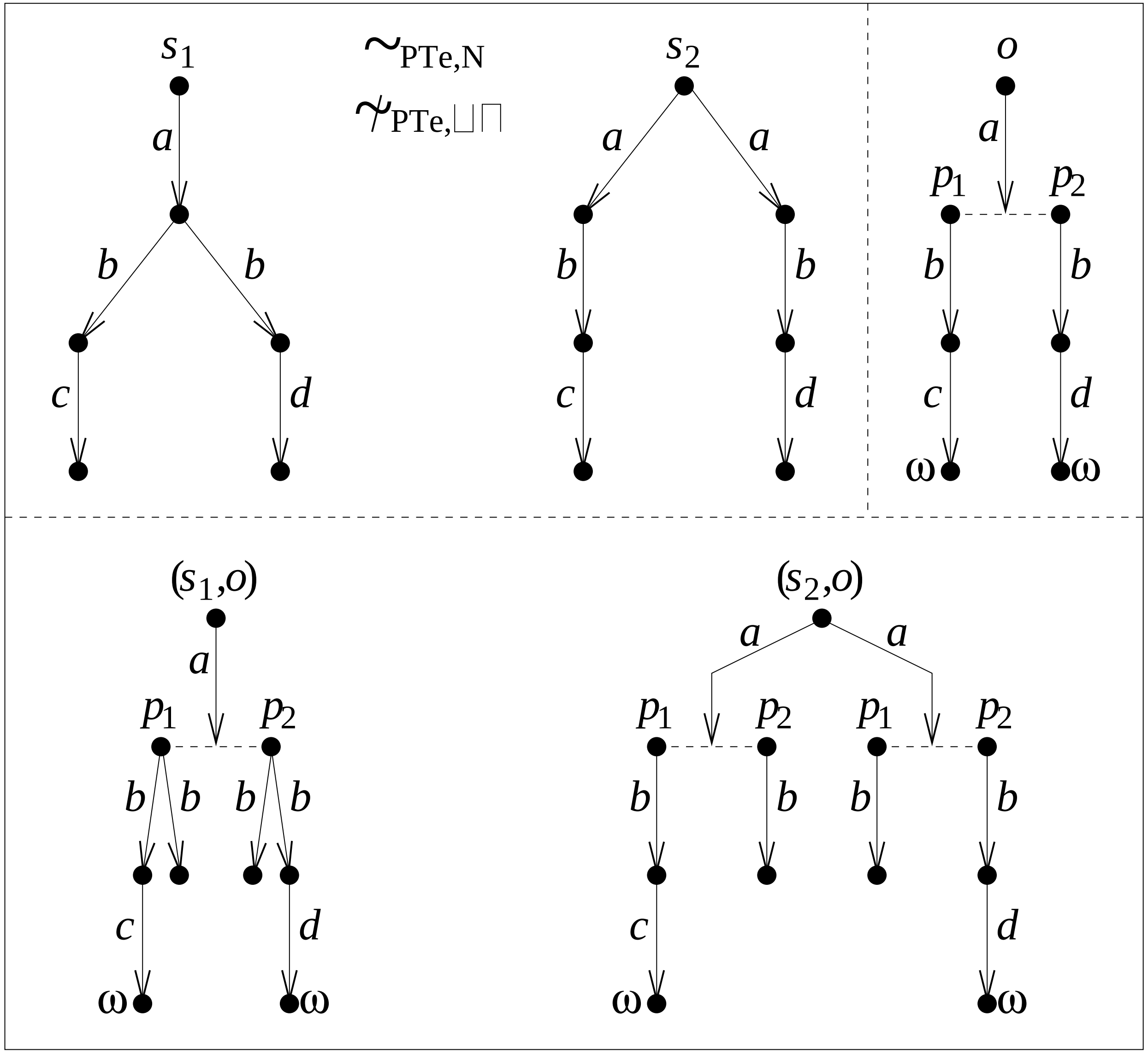
- $s_1 \sim_{\mathrm{PTe,N}} s_2$ iff for every NPT $\mathcal{T} = (O, A, \longrightarrow_{\mathcal{T}})$ with initial state $o \in O$ and for all $\alpha \in A^*$ it holds that for each $\mathcal{Z}_1 \in Res_{\max,\mathcal{C},\alpha}(s_1, o)$ there exists $\mathcal{Z}_2 \in Res_{\max,\mathcal{C},\alpha}(s_2, o)$ such that:

$$prob(\mathcal{SCC}(z_{s_1,o}, \alpha)) \;=\; prob(\mathcal{SCC}(z_{s_2,o}, \alpha))$$

and symmetrically for each $\mathcal{Z}_2 \in Res_{\max,\mathcal{C},\alpha}(s_2, o)$.



s1
~PTe,⊔⊓
≁PTe,N
s2
o
a
a
a
0.5
0.5
b
b
c
c
a
a
b
c
a
0.5
0.5
b
c
ω
(s1,o)
(s2,o)
a
a
a
a
a
0.5
0.5
0.25
0.25
0.25
0.25
0.5
0.5
0.5
0.5
0.5
0.5
b
b
c
c
b
c
ω
ω
ω

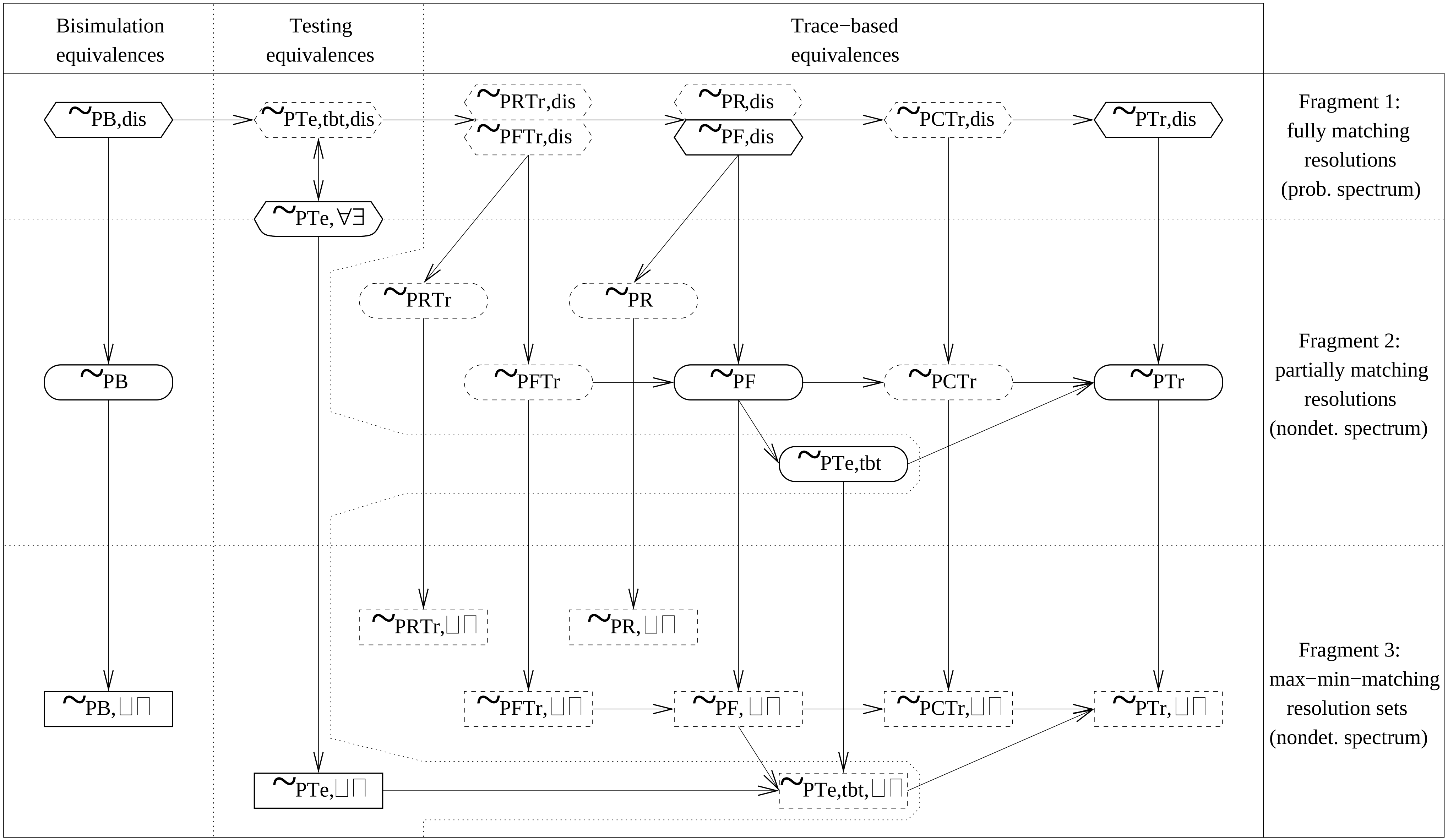


s_1
$\sim_{PTe,N}$
$\not\sim_{PTe,\sqcup\sqcap}$
s_2
o
a b b c d
a a b b c d
a p_1 p_2 b b c d ω ω
(s_1,o)
a p_1 p_2 b b b b c d ω ω
(s_2,o)
a a p_1 p_2 p_1 p_2 b b b b c d ω ω

Spectrum of NPLTS Behavioral Equivalences

- Spectrum of LTS behavioral equivalences in [Van Glabbeek, 1990] and of GPLTS behavioral equivalences in [Jou-Smolka, 1990].

- Three different fragments in the spectrum for NPLTS models:
  - $\sim_{\mathrm{PB,dis}}$ and $\sim_{\mathrm{PTr,dis}}$ require fully matching resolutions: for every trace, the probability of performing that trace must be the same in both resolutions, which thus possess the same trace distribution.
  - $\sim_{\mathrm{PB}}$ and $\sim_{\mathrm{PTr}}$ allow for partially matching resolutions: a resolution on one side is allowed to be matched by different resolutions on the other side with respect to different traces.
  - $\sim_{\mathrm{PTe},\sqcup\sqcap}$ considers only extremal probabilities over all resolutions.


Bisimulation equivalences
Testing equivalences
Trace−based equivalences
∼PB,dis
∼PTe,tbt,dis
∼PRTr,dis
∼PFTr,dis
∼PR,dis
∼PF,dis
∼PCTr,dis
∼PTr,dis
∼PTe,∀∃
Fragment 1: fully matching resolutions (prob. spectrum)
∼PRTr
∼PR
∼PB
∼PFTr
∼PF
∼PCTr
∼PTr
∼PTe,tbt
Fragment 2: partially matching resolutions (nondet. spectrum)
∼PRTr,⊔⊓
∼PR,⊔⊓
∼PB,⊔⊓
∼PFTr,⊔⊓
∼PF,⊔⊓
∼PCTr,⊔⊓
∼PTr,⊔⊓
∼PTe,⊔⊓
∼PTe,tbt,⊔⊓
Fragment 3: max−min−matching resolution sets (nondet. spectrum)

Future Work

- Defining an ULTRAS-based operational semantics of process calculi of nondeterministic, probabilistic, stochastic, or mixed nature: general properties & relative expressiveness.
- Studying a generic process algebra together with uniform results for congruence properties and equational/logical characterizations.
- Providing uniform definitions of weak behavioral equivalences.
- Including richer/different models: Markov automata, timed automata, probabilistic timed automata, and stochastic automata.
- Extension of ULTRAS with transitions of the form $\mathcal{D} \xrightarrow{a} \mathcal{D}'$:
  - State distributions describing alternatives among global states: Kleisli lifting of state-to-state-distribution reachability relations.
  - State distributions describing combinations of local states: Petri nets as $\mathbb{N}$-ULTRAS models in which states are Petri net places and transitions are Petri net transitions.