Weak Bisimulation and Causality

Joint work with Davide Sangiorgi

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Outline

Some drawbacks of weak bisimulation

Some Ideas

(Partial) Results

Problems and Further Works

Weak Bisimulation Game

A relation $\mathcal{R}$ s.t

$$
\begin{array}{ccc}
P & \mathcal{R} & Q \\
{\scriptstyle\alpha}\downarrow & & \Downarrow{\scriptstyle\alpha} \\
P' & \mathcal{R} & Q'
\end{array}
\qquad\qquad
\begin{array}{ccc}
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$$
\begin{array}{ccc}
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P & \mathcal{R} & Q \\
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Figure: Weak Bisimulation Game.

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...and their related problems:

- Undesiderable distinctions.

Example (Atomic vs gradual commitment)

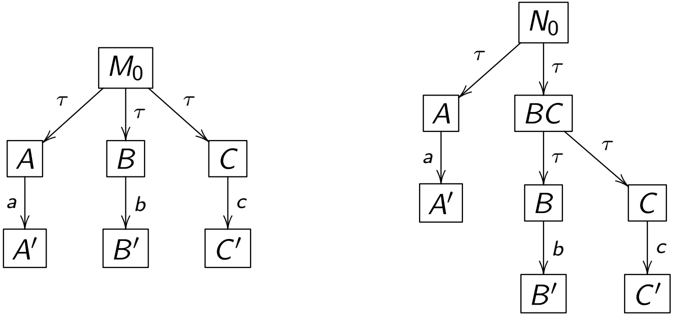

Figure: Implementation and Specification of a Multi-way Synchronization

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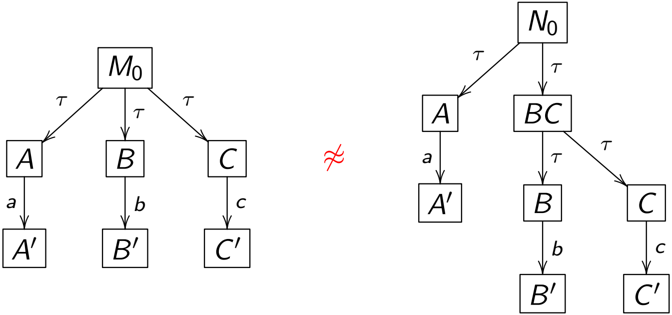

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- Expensive algorithms.
- Failure of a number of up-to techniques.

  E.g. 'Up-to $\approx$' unsound.

Remark

Removing the bisimulation game on τ-actions

$\Downarrow$

no congruence relation.

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Moreover, the resulting relation does not solve the atomic vs gradual commitment problem:

$$M_0 \mid d \not\approx_\tau N_0 \mid d.$$

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Abstract from τ-actions necessary to release a visible action.

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$\leadsto$ A partial account to causality.

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Formalize causality using colours:

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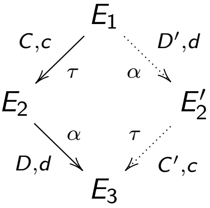
Proposition (Causality)

If $E_1 \xrightarrow{\tau}_{C,c} E_2 \xrightarrow{\alpha}_{D,d} E_3$ *and* $c \notin D$, *then we can permute the transitions*



with $C' \cup D' = C \cup D$.

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Definition (Partially-causal Bisimulation)

Whenever $P \mathcal{R} Q$:

- $P \overset{\alpha}{\rightsquigarrow} P'$ implies $Q \overset{\alpha}{\rightsquigarrow} Q'$ and $P' \mathcal{R} Q'$.
- And the converse.

$$\sim_{pc} = \bigcup \{\mathcal{R} \mid \mathcal{R} \text{ pc-bisimulation}\}.$$

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- Every weak transition $P \stackrel{\alpha}{\Longrightarrow} Q$ can be decomposed as:

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- Neither a true nor interleaving semantics:

$$a \mid b \quad \sim_{pc} \quad a.b + b.a$$

$a \mid \tau$	no causal relation between τ and a
$a.\tau + \tau.a$	causal relation between τ and a.

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Theorem

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Adding Stability

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Definition (Stable pc-bisimulation)

Whenever $P \mathcal{R} Q$:

- $P \stackrel{\alpha}{\leadsto} P'$ implies $Q \stackrel{\alpha}{\leadsto} Q'$ and $P' \mathcal{R} Q'$.
- $P \Longrightarrow P'$ and P' is stable, implies $Q \Longrightarrow Q'$ and $P' \mathcal{R} Q'$.
- And the converse.

$$\sim_{spc} = \bigcup \{\mathcal{R} \mid \mathcal{R} \text{ stable pc-bisimulation}\}.$$

Adding Stability

On diverge-free LTSs we can use spc-bisimilarity to check testing equivalence:

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- Other equivalences to be considered?
- Up-to techniques still to be investigated.

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Thanks for the attention!